COMPOSITION of Functions

The **Composition** of functions is when you replace the variable of a function by another function.

Example: If $f(x) = \frac{3x-5}{x+1}$ and $g(x) = x^3$

Then $f(g(x)) = \frac{3x^3-5}{x^3+1}$

Notation: $f \circ g(x) = f(g(x))$

Examples: If $f(x) = 5x$, $g(x) = x - 3$ and $h(x) = x^2$

a) $g \circ f(x) =$ $g(5x) = 5x - 3$

b) $h \circ f(x) =$ $h(5x) = (5x)^2 = 25x^2$

c) $g \circ h(x) =$ $g(x^2) = x^2 - 3$

d) $h \circ g(x) =$ $h(x-3) = (x-3)^2 = x^2 - 6x + 9$

e) $f \circ h(3) =$ $f(9) = 5 \times 9 = 45$

f) $f \circ f(x) =$ $f(5x) = 5(5x) = 25x$

f) $g \circ g(-2) =$ $g(-5) = -5 - 3 = -8$

Your turn: If $f(x) = x + 6$ and $g(x) = -3x + 5$

a) $g \circ f(x) =$ $g(x+6) = -3(x+6) + 5 = -3x - 18 + 5 = -3x - 13$

b) $f \circ g(x) =$ $f(-3x+5) = -3x + 5 + 6 = -3x + 11$

c) $f \circ f(x) =$ $f(x+6) = x + 6 + 6 = x + 12$

d) $f \circ g(-4) =$ $f(17) = 17 + 6 = 23$ because $g(-4) = 17$

e) $g \circ f(0) =$ $g(6) = -3 \times 6 + 5 = -13$

f) $g \circ g(8) =$ $g(-19) = -3(-19) + 5 = 62$

Decomposing a function: It is often useful to see what a function is "made of" in terms of usual functions.

Example Write the following functions as a composite function.

a) $h(x) = (x-2)^2 + (x-2) + 1$

↳ $f(x) = x - 2$

$g(x) = x^2 + x + 1$

$h(x) = g \circ f(x)$

b) $k(x) = \sqrt{x^3 + 1}$

↳ $f(x) = x^3 + 1$

$g(x) = \sqrt{x}$

$k(x) = g \circ f(x)$

⚠ Sometimes there are several possible answers... just pick one!

Your turn : Decompose the following functions in terms of usual functions :

a) $f(x) = \sqrt{3x+1}$

$g(x) = \sqrt{x}$
$h(x) = 3x+1$

$f(x) = g \circ h(x)$

b) $g(x) = (3x+5)^2$

$h(x) = x^2$
$k(x) = 3x+5$

$g(x) = h \circ k(x)$

c) $h(x) = \sqrt{x^2-4}$

$f(x) = \sqrt{x}$
$g(x) = x^2 - 4$

$h(x) = f \circ g(x)$

d) $j(x) = |2x-5|$

$f(x) = 2x-5$
$g(x) = |x|$

$j(x) = g \circ f(x)$

e) $k(x) = 5\sqrt{2x+3} - 2$

$f(x) = 5x - 2$
$g(x) = \sqrt{x}$
$h(x) = 2x+3$

$k(x) = f \circ g \circ h(x)$

f) $l(x) = \sqrt{|x^2+3x-5|}$

$f(x) = \sqrt{x}$
$g(x) = |x|$
$h(x) = x^2 + 3x - 5$

$l(x) = f \circ g \circ h(x)$