

## COMPOSITION of Functions

The **Composition** of functions is when you replace the variable of a function by another function.

Example: If  $f(x) = \frac{3x-5}{x+1}$  and  $g(x) = x^3$

$$\text{Then } f(g(x)) = \frac{3x^3-5}{x^3+1}$$

Notation:  $f \circ g(x) = f(g(x))$

Examples: If  $f(x) = 5x$ ,  $g(x) = x - 3$  and  $h(x) = x^2$

- a)  $g \circ f(x) = g(5x) = 5x - 3$
- b)  $h \circ f(x) = h(5x) = (5x)^2 = 25x^2$
- c)  $g \circ h(x) = g(x^2) = x^2 - 3$
- d)  $h \circ g(x) = h(x-3) = (x-3)^2 = x^2 - 6x + 9$
- e)  $f \circ h(3) = f(9) = 5 \times 9 = 45$
- f)  $f \circ f(x) = f(5x) = 5(5x) = 25x$
- f)  $g \circ g(-2) = g(-5) = -5 - 3 = -8$

Your turn: If  $f(x) = x + 6$  and  $g(x) = -3x + 5$

- a)  $g \circ f(x) =$
- b)  $f \circ g(x) =$
- c)  $f \circ f(x) =$
- d)  $f \circ g(-4) =$
- e)  $g \circ f(0) =$
- f)  $g \circ g(8) =$

**Decomposing a function:** It is often useful to see what a function is "made of" in terms of usual functions.

Example Write the following functions as a composite function.

a)  $h(x) = (x-2)^2 + (x-2) + 1$

b)  $k(x) = \sqrt{x^3+1}$

$\hookrightarrow f(x) = x - 2$   
 $g(x) = x^2 + x + 1$

$\hookrightarrow f(x) = x^3 + 1$   
 $g(x) = \sqrt{x}$

$h(x) = g \circ f(x)$

$k(x) = g \circ f(x)$

⚠ Sometimes there are several possible answers... just pick one!

Your turn: Decompose the following functions in terms of usual functions :

a)  $f(x) = \sqrt{3x + 1}$

b)  $g(x) = (3x + 5)^2$

c)  $h(x) = \sqrt{x^2 - 4}$

d)  $j(x) = |2x - 5|$

e)  $k(x) = 5\sqrt{2x + 3} - 2$

f)  $l(x) = \sqrt{|x^2 + 3x - 5|}$