

USUAL EQUATIONS - worksheet

1. Solve.

a) $2 - 3(x + 1) = 5(x + 3)$

$$2 - 3x - 3 = 5x + 15$$

$$-16 = 8x$$

$$\boxed{x = -2}$$

b) $(2x + 3)(x - 5) = 2x^2 - 6x + 1$

$$2x^2 - 10x + 3x - 15 = 2x^2 - 6x + 1$$

$$-7x - 15 = -6x + 1$$

$$\boxed{-16 = x}$$

c) $\frac{3}{7}x - 5 = \frac{2}{3}(x + 1)$

$$\frac{3}{7}x - 5 = \frac{2}{3}x + \frac{2}{3}$$

$$9x - 105 = 14x + 14$$

$$-119 = 5x$$

$$\boxed{x = -\frac{119}{5}}$$

d) $\frac{5(x-3)}{6} = \frac{2}{3}(x+2)$

$$5(x-3) = 4(x+2)$$

$$5x - 15 = 4x + 8$$

$$\boxed{x = +23}$$

2. Solve (try to vary the methods used):

a) $x^2 + 5x + 4 = 0$

$$(x+4)(x+1) = 0$$

$$\boxed{x = -4 \text{ \& } x = -1}$$

b) $6x^2 - 7x - 5 = 0$

$$\Delta = 169$$

$$x = \frac{7 \pm 13}{12}$$

$$\boxed{-\frac{1}{2}}$$

$$\boxed{\frac{5}{3}}$$

c) $3(x-5)^2 - 27 = 0$

$$3(x-5)^2 = 27$$

$$(x-5)^2 = 9$$

$$x-5 = \pm 3$$

$$x = 5 \pm 3 \rightarrow \begin{cases} \boxed{2} \\ \boxed{8} \end{cases}$$

d) $(2x-1)(3x+2) = 1$

$$6x^2 + 4x - 3x - 2 - 1 = 0$$

$$6x^2 + x - 3 = 0$$

$$\Delta = 1 - 4(6)(-3) = 73$$

$$\boxed{x = \frac{-1 \pm \sqrt{73}}{12}}$$

← exact value

3. Solve algebraically

a) $|x^2 - 2x| = 1$

$$\begin{aligned} x^2 - 2x &= 1 \quad \text{or} \quad -x^2 + 2x = 1 \\ x^2 - 2x - 1 &= 0 & x^2 - 2x + 1 &= 0 \\ \Delta = 4 + 4 = 8 & & (x-1)^2 &= 0 \\ x = \frac{2 \pm \sqrt{8}}{2} &= 1 \pm \sqrt{2} & x &= 1 \end{aligned}$$

TESTS --- or restr

3 Sol: $1 \pm \sqrt{2}$ & 1

c) $\frac{4x-1}{x+2} - \frac{x+1}{x-2} = \frac{x^2-4x+24}{x^2-4}$

• Restr: $|D = \mathbb{R} \setminus \{\pm 2\}$

$$\begin{aligned} (4x-1)(x-2) - (x+1)(x+2) &= x^2 - 4x + 24 \\ 4x^2 - 8x - x + 2 - (x^2 + 2x + x + 2) &= x^2 - 4x + 24 \end{aligned}$$

$$\begin{aligned} 4x^2 - 9x + 2 - x^2 - 3x - 2 - x^2 + 4x - 24 &= 0 \\ 2x^2 - 8x - 24 &= 0 \\ x^2 - 4x - 12 &= 0 \\ (x-6)(x+2) &= 0 \\ \boxed{x=6} \quad \cancel{x=-2} \text{ restr} \end{aligned}$$

e) $4 + \sqrt{x-1} = x-3$

• Restr: $x-1 \geq 0$ $\underline{x \geq 1}$

$$\begin{aligned} \sqrt{x-1} &= x-7 \\ x-1 &= x^2 - 14x + 49 \\ x^2 - 15x + 50 &= 0 \\ (x-5)(x-10) &= 0 \\ x=5 \quad x=10 \end{aligned}$$

TESTS :

$4 + \sqrt{x-1} = x-3$	$4 + \sqrt{x-1} = x-3$
$4 + \sqrt{4} \quad \quad 5-3$	$4 + \sqrt{9} \quad \quad 10-3$
$6 \quad \quad 2$	$7 \quad \quad 7 \checkmark$
$6 \quad \quad 2_x$	

Sol: $\boxed{x=10}$

b) $|x+5| = 4x-1$

$$\begin{aligned} x+5 &= 4x-1 & \text{or} & \quad -x-5 = 4x-1 \\ 6 &= 3x & & \quad -4 = 5x \\ x &= 2 & & \quad x = -\frac{4}{5} \end{aligned}$$

TESTS $x=2 \checkmark$ $x = -\frac{4}{5} \times$
or...

Sol: $\{2\}$

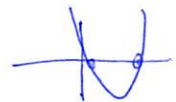
d) $\frac{x^2+x+2}{x+1} - x = \frac{x^2-5}{x^2-1}$

• Restr: $|x \neq \pm 1$

$$\begin{aligned} (x^2+x+2)(x-1) - x(x+1)(x-1) &= x^2-5 \\ x^3 - x^2 + x^2 - x + 2x - 2 - x(x^2-1) &= x^2-5 \\ x^3 + x - 2 - x^3 + x &= x^2-5 \\ x^2 - 2x - 3 &= 0 \\ (x-3)(x+1) &= 0 \\ \boxed{x=3} \quad \cancel{x=-1} \end{aligned}$$

f) $\sqrt{x^2 - 10x + 1} = 5$

• Restr: $x^2 - 10x + 1 \geq 0$



$x \leq 5 - 2\sqrt{6}$ or $x \geq 5 + 2\sqrt{6}$

$$\begin{aligned} x^2 - 10x + 1 &= 25 \\ x^2 - 10x - 24 &= 0 \\ (x-12)(x+2) &= 0 \\ x=12 \quad x=-2 \end{aligned}$$

TESTS \checkmark and \checkmark

Sol: $\boxed{-2 \text{ \& } 12}$