Solutions

INVERSE OF A FUNCTION – Exponentials and Logarithms

To get an inverse function, we usually exchange x and y and solve for the new y…

Examples: Determine the inverse of the following functions:

1) $y = \log_3 x$

$\hookrightarrow x = \log_3 y$

$\boxed{3^x = y}$

2) $y = 5^x$

$\hookrightarrow x = 5^y$

$\boxed{\log_5 x = y}$

3) $y = 2\log_5(3x + 1)$

$\hookrightarrow x = 2\log_5(3y+1)$

$\frac{x}{2} = \log_5(3y+1)$

$5^{x/2} = 3y+1$

$3y = 5^{x/2} - 1$

$\boxed{y = \frac{1}{3}\cdot 5^{x/2} - \frac{1}{3}}$

4) $y = 2 \times e^{x-4} + 1$

$\hookrightarrow x = 2 \times e^{y-4} + 1$

$\frac{x-1}{2} = e^{y-4}$

$\ln\left(\frac{x-1}{2}\right) = y - 4$

$\boxed{y = \ln\left(\frac{x-1}{2}\right) + 4}$

5) $y = \frac{2}{3}\log_3(2x + 4) - 3$

$\hookrightarrow x = \frac{2}{3}\log_3(2y+4) - 3$

$x + 3 = \frac{2}{3}\log_3(2y+4)$

$\frac{3}{2}(x+3) = \log_3(2y+4)$

$3^{\frac{3}{2}(x+3)} = 2y + 4$

$2y = 3^{\frac{3}{2}(x+3)} - 4$

$\boxed{y = \frac{1}{2}\cdot 3^{\frac{3}{2}(x+3)} - 2}$

6) $y = \frac{3}{4}5^{4x-12} - 1$

$\hookrightarrow x = \frac{3}{4}\cdot 5^{4y-12} - 1$

$x + 1 = \frac{3}{4}\cdot 5^{4y-12}$

$\frac{4}{3}(x+1) = 5^{4y-12}$

$\log_5\left(\frac{4}{3}(x+1)\right) = 4y - 12$

$4y = \log_5\left(\frac{4}{3}(x+1)\right) + 12$

$\boxed{y = \frac{1}{4}\log_5\left(\frac{4}{3}(x+1)\right) + 3}$

Notice how the transformations are transferred to the inverse functions…