Logarithms – Open-ended Solutions

1. **a)** $\dfrac{\left(3^{\frac{1}{5}}\right)^{10}\left(3^{-3}\right)}{9} = \dfrac{3^2\cdot 3^{-3}}{3^2} = 3^{-3} = \dfrac{1}{3^3} = \dfrac{1}{27}$

b) $\dfrac{\left(-4x^2y^{-2}\right)^{-3}}{x^{-1}y^2} = \dfrac{(-4)^{-3}x^{-6}y^6}{x^{-1}y^2} = \dfrac{-y^4}{64x^5}$

c) $\dfrac{125^{3x-1}\cdot 25^{1-2x}}{\left(\frac{1}{5}\right)^{2x-3}} = \dfrac{5^{3(3x-1)}\cdot 5^{2(1-2x)}}{5^{-1(2x-3)}} = \dfrac{5^{9x-3}\cdot 5^{2-4x}}{5^{-2x+3}} = 5^{9x-3+2-4x+2x-3} = 5^{7x-4}$

d) $\dfrac{2x^4\cdot 3^{5x} - 4x^3\cdot 3^{5x}}{x^3 - 2x^2} = \dfrac{2x^3\cdot 3^{5x}(x-2)}{x^2(x-2)} = 2x\cdot 3^{5x}$

e) $\left(4^{-x}\cdot 8^x\right)^2 = 4^{-2x}\cdot 8^{2x} = \left(2^2\right)^{-2x}\cdot\left(2^3\right)^{2x} = 2^{-4x}\cdot 2^{6x} = 2^{-4x+6x} = 2^{2x} = 4^x$

f) $\dfrac{2^x\left(2^x+2^{-x}\right) - 2^x\left(2^x-2^{-x}\right)}{2^{-2}} = \dfrac{2^{2x}+2^0-2^{2x}+2^0}{2^{-2}} = \dfrac{1+1}{2^{-2}} = 2\cdot 2^2 = 2^3 = 8$

2. **a)** $4^{x^2-x} = 1 \rightarrow 4^{x^2-x} = 4^0 \rightarrow x^2 - x = 0 \rightarrow x(x-1) = 0 \rightarrow x = 0, 1$

b) $3^{x^2} = 9\cdot 3^{-x} \rightarrow 3^{x^2} = 3^2\cdot 3^{-x} \rightarrow 3^{x^2} = 3^{-x+2} \rightarrow x^2 = -x+2 \rightarrow x^2+x-2 = 0 \rightarrow$
$(x+2)(x-1) = 0 \rightarrow x = -2\ ,\ 1$

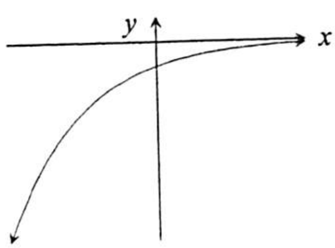
c) $4^{\sqrt{x+1}} = 2^{3x-2} \rightarrow 2^{2\sqrt{x+1}} = 2^{3x-2} \rightarrow 2\sqrt{x+1} = 3x-2 \rightarrow \left(2\sqrt{x+1}\right)^2 = (3x-2)^2 \rightarrow$
$4(x+1) = 9x^2 - 12x + 4 \rightarrow 9x^2 - 16x = 0 \rightarrow x(9x-16) = 0 \rightarrow x = 0, \dfrac{16}{9}$ Check, reject 0, $\therefore\ x = \dfrac{16}{9}$

d) $4^{-|x+1|} = \dfrac{1}{16} \rightarrow 4^{-|x+1|} = 4^{-2} \rightarrow -|x+1| = -2 \rightarrow |x+1| = 2 \rightarrow x+1 = 2$ or $x+1 = -2 \rightarrow x = 1\ ,\ -3$

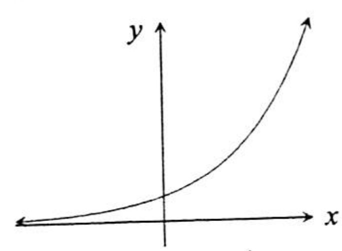
e) $4^{-2x+1} = 8^{x-4} \rightarrow 2^{2(-2x+1)} = 2^{3(x-4)} \rightarrow 2^{-4x+2} = 2^{3x-12} \rightarrow -4x+2 = 3x-12 \rightarrow -7x = -14 \rightarrow x = 2$

f) $9^{2x-1} = \left(\dfrac{1}{27}\right)^{x+2} \rightarrow 3^{2(2x-1)} = 3^{-3(x+2)} \rightarrow 2(2x-1) = -3(x+2) \rightarrow 4x-2 = -3x-6 \rightarrow 7x = -4 \rightarrow x = -\dfrac{4}{7}$

3. **a)** $y = -ab^x = -f(x)$ will reflect the graph over the x-axis.



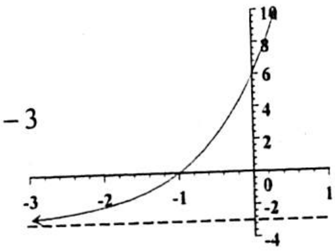
b) $y = ab^{-x} = f(-x)$ will reflect the graph over the y-axis.



4. a) Graph $y=3^x$ is shifted left two units and down three units.

Domain: all real numbers
Range: $y > -3$
x-intercept: -1
y-intercept: 6
Asymptote: $y = -3$

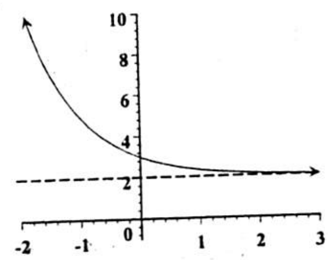
$y=3^{x+2}-3$



b) Graph $y=3^x$ is reflected about the y-axis and up two units.

Domain: all real numbers
Range: $y > 2$
x-intercept: no x-intercept
y-intercept: 3
Asymptote: $y = 2$

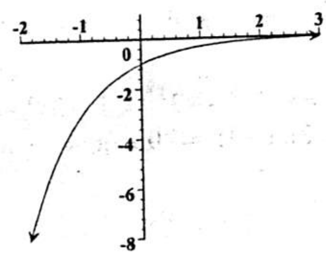
$y=(\frac{1}{3})^x+2$



c) Graph of $y=3^x$ is reflected about both the x-axis and y-axis.

Domain: all real numbers
Range: y < 0
x-intercept: no x-intercept
y-intercept: -1
Asymptote: $y = 0$

$y=-3^{-x}$



5. a) $y=b^x \to 3=b^{-1} \to b=\frac{1}{3}$

b) $y=b^x \to 27=b^{\frac{3}{2}} \to b=27^{\frac{2}{3}} \to b=(3^3)^{\frac{2}{3}} \to b=3^2 \to b=9$

c) $y=b^x \to \frac{1}{9}=b^{-\frac{2}{3}} \to b=\left(\frac{1}{9}\right)^{-\frac{3}{2}} \to b=\left(3^{-2}\right)^{-\frac{3}{2}} \to b=3^3 \to b=27$

6. $y=c\cdot 2^{kx} \to 4=c\cdot 2^{k\cdot 0} \to c=4 \to y=4\cdot 2^{kx} \to 256=4\cdot 2^{12k} \to 64=2^{12k} \to$
$2^6=2^{12k} \to 12k=6 \to k=\frac{1}{2}$, therefore, $y=4\cdot 2^{\frac{1}{2}x}$ or $y=2^{\frac{1}{2}x+2}$

7. a) $\frac{10^{8.9}}{10^{6.4}}=10^{2.5}$ = 316 times as strong

b) 1 000 = 10^3. A 4.9 earthquake is a $10^{4.9}$ measure. So the San Francisco earthquake has a $10^{4.9}\cdot 10^3=10^{4.9+3}=10^{7.9}$ or a Richter scale measure of 7.9

c) $A=P(1+\frac{r}{n})^{nt} \to A=1000(1+\frac{0.06}{4})^{4\times 8} \to A=1000(1.015)^{32} \to A$ = \$1 610.32

d) $A=A_0(x)^{\frac{t}{T}} \to A=84(\frac{1}{2})^{\frac{23}{4}} \to A$ = 1.56 grams of argon-39 remains

e) $A=P(1+\frac{r}{n})^{nt} \to A = 12\,250(1+\frac{0.096}{12})^{12\times 10}$ = \$31 871.31

f) $A=P(1+\frac{r}{n})^{nt} \to A=30\,000\,000(1+\frac{0.019}{1})^{1\times 32} \to A=30\,000\,000(1.019)^{32}$ = 54 789 223 =
55 million to the nearest million.

a) $2^{-3}=\frac{1}{8} \rightarrow \log_2 \frac{1}{8}=-3$

b) $2^x=5 \rightarrow \log_2 5=x$

c) $x^{1.2}=4.7 \rightarrow \log_x 4.7=1.2$

d) $a^b=c \rightarrow \log_a c=b$

9. a) $\log_x 32=5 \rightarrow x^5=32 \rightarrow x^5=2^5 \rightarrow x=2$

b) $\log_{\sqrt{2}} 16=x \rightarrow \sqrt{2}^x=16 \rightarrow 2^{\frac{1}{2}x}=2^4 \rightarrow \frac{1}{2}x=4 \rightarrow x=8$

c) $\log_{\sqrt{3}} x=6 \rightarrow \sqrt{3}^6=x \rightarrow x=\left(3^{\frac{1}{2}}\right)^6 \rightarrow x=3^3 \rightarrow x=27$

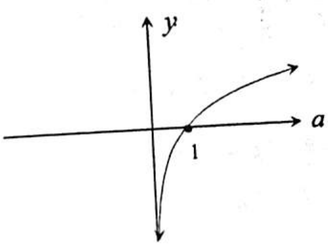
d) $\log_{3x} 36=2 \rightarrow (3x)^2=36 \rightarrow 9x^2=36 \rightarrow x^2=4 \rightarrow x=\pm 2$, reject $x=-2$ $\therefore$ $x=2$

10. a) $y=5^x$ and $y=\log_5 x$ are the inverse of each other
Therefore, if (a, b) is a point on $y=5^x$ then (b, a) must be a point on $y=\log_5 x$
Two other points on $y=\log_5 x$ are $(5, 1)$ and $(1, 0)$.

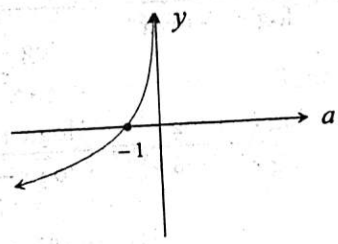
b) $f(x)=\log_2 x \rightarrow -f(x)=-\log_2 x : -f(x)$ is reflected over the x-axis.
So putting a negative in front of a logarithmic statement reflects the equation over the x-axis.
The point $(1, 0)$ is on the x-axis so the point will not change, therefore, the answer is $(1, 0)$.

c) $y=\log_{\frac{1}{b}} a=\dfrac{\log a}{\log \frac{1}{b}}=\dfrac{\log a}{\log 1-\log b}=\dfrac{\log a}{-\log b}=-\log_b a$ so if (c, d) is a point on the graph $y=\log_b a$ then $(c, -d)$ must be on graph $y=\log_{\frac{1}{b}} a$. Two other points on $\log_{\frac{1}{b}} a$ are $\left(\frac{1}{b}, 1\right)$ and $(1, 0)$

d) (*i*) $y=-\log_b a=-f(a)$ will reflect the graph over the x-axis



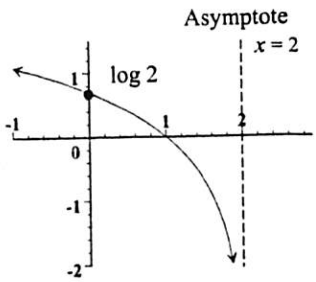
(*ii*) $y=\log_b(-a)=f(-a)$ will reflect the graph over the y-axis



(*iii*) From question **c**, above, $y=\log_{\frac{1}{b}} a=-\log_b a$, which makes the graph the same as **d**(*i*), above.

e) $y=\log_{x+1}(x-2)$ The base $x+1>0$ and $x+1\neq 1$ $\therefore$ $x>-1$ and $x\neq 0$.
The log term is $(x-2)>0$ so $x>2$.
Take the intersection of $x>-1, x\neq 0$ with $x>2$ which is $x>2$.
Therefore, the domain of $y=\log_{x+1}(x-2)$ is $x>2$.

10. f) Graph $y = \log(2-x)$



11. a) $y = 8^{x-2} \;\to\; f^{-1}: x = 8^{y-2} \;\to\; \log_8 x = y-2 \;\to\; y = 2+\log_8 x \;\to\; f^{-1}(x) = 2 + \log_8 x$

b) $f(x) = 5^{4x-1}+6 \to f^{-1}:\ x = 5^{4y-1}+6 \to x-6 = 5^{4y-1} \to \log_5(x-6) = 4y-1 \to$
$4y = \log_5(x-6)+1 \to f^{-1}(x) = \frac{1}{4}\log_5(x-6)+\frac{1}{4}$

c) $f: y+1 = \log_3(x-2) \;\to f^{-1}:\ x+1 = \log_3(y-2) \to y-2 = 3^{x+1} \to$
$y = 3^{x+1}+2 \;\to\; f^{-1}(x) = 3^{x+1}+2$

d) $f(x) = 2+\log(5x-3) \to f^{-1}(x):\ x = 2+\log(5y-3) \to x-2 = \log(5y-3) \to$
$5y-3 = 10^{x-2} \to 5y = 10^{x-2}+3 \to y = \dfrac{10^{x-2}+3}{5} \;\to\; f^{-1}(x) = \dfrac{10^{x-2}+3}{5}$

12. a) $\log_b x^{\log_x a} = \log_x a \cdot \log_b x = \dfrac{\log a}{\log x}\cdot\dfrac{\log x}{\log b} = \dfrac{\log a}{\log b} = \log_b a$

b) $x^{\log_x 20 - \log_x 4} = x^{\log_x \frac{20}{4}} = x^{\log_x 5} = 5$

c) $(\log_2 10)(\log 48 - \log 3) = \dfrac{\log 10}{\log 2}\cdot\log 16 = \dfrac{1}{\log 2}\cdot\log 2^4 = \dfrac{4\log 2}{\log 2} = 4$

d) <u>Method 1</u>: $\dfrac{\log x^3+\log x^5}{\log x^6-\log x^3} = \dfrac{\log x^3\cdot x^5}{\log\dfrac{x^6}{x^3}} = \dfrac{\log x^8}{\log x^3} = \dfrac{8\log x}{3\log x} = \dfrac{8}{3}$

<u>Method 2</u>: $\dfrac{\log x^3+\log x^5}{\log x^6-\log x^3} = \dfrac{3\log x+5\log x}{6\log x-3\log x} = \dfrac{8\log x}{3\log x} = \dfrac{8}{3}$

e) $\left(\dfrac{a}{b}\right)^{\log 0.5}\cdot\left(\dfrac{a}{b}\right)^{\log 0.2} = \left(\dfrac{a}{b}\right)^{\log 0.5+\log 0.2} = \left(\dfrac{a}{b}\right)^{\log(0.5)(0.2)} = \left(\dfrac{a}{b}\right)^{\log 0.1} = \left(\dfrac{a}{b}\right)^{-1} = \dfrac{b}{a}$

f) <u>Method 1</u>: $4^{-2\log_4 3} = x \to \log_4 x = -2\log_4 3 \to \log_4 x = \log_4 3^{-2} \to x = 3^{-2} \to x = \frac{1}{9}$

<u>Method 2</u>: $4^{\log_4 3^{-2}} = 3^{-2} \to x = \frac{1}{9}$. (rule #7, page 48)

g) $10\log_4 x - 12\log_8 x = \dfrac{10\log x}{\log 4} - \dfrac{12\log x}{\log 8} = \dfrac{10\log x}{\log 2^2} - \dfrac{12\log x}{\log 2^3} = \dfrac{10\log x}{2\log 2} - \dfrac{12\log x}{3\log 2} =$
$5\log_2 x - 4\log_2 x = \log_2 x$

h) Method 1: $\log \pi + \log \frac{\sqrt{2}}{\pi} + \frac{1}{2}\log \frac{3}{2} - \log \frac{\sqrt{3}}{10} = \log\left(\frac{\pi \cdot \frac{\sqrt{2}}{\pi} \cdot \left(\frac{3}{2}\right)^{\frac{1}{2}}}{\frac{\sqrt{3}}{10}}\right) \rightarrow \log\left(\frac{3^{\frac{1}{2}}}{\frac{\sqrt{3}}{10}}\right) = \log 10 = 1$

Method 2: $\log \pi + \log\sqrt{2} - \log \pi + \log\sqrt{3} - \log\sqrt{2} - \log\sqrt{3} + \log 10 = \log 10 = 1$

i) $\log(1-x^3) - \log(1+x+x^2) - \log(1-x) = \log\frac{(1-x^3)}{(1+x+x^2)(1-x)} = \log\frac{(1-x)(1+x+x^2)}{(1+x+x^2)(1-x)} = \log 1 = 0$

j) $\frac{\log_a x}{\log_{ab} x} - \frac{\log_a x}{\log_b x} = \frac{\frac{\log x}{\log a}}{\frac{\log x}{\log ab}} - \frac{\frac{\log x}{\log a}}{\frac{\log x}{\log b}} = \frac{\log ab}{\log a} - \frac{\log b}{\log a} = \frac{\log a + \log b - \log b}{\log a} = 1$

k) $\frac{1}{\log_a x} + \frac{1}{\log_b x} = \log_x a + \log_x b = \log_x ab$

l) $(\log_5 9)(\log_3 7)(\log_7 5) = \frac{\log 9}{\log 5} \cdot \frac{\log 7}{\log 3} \cdot \frac{\log 5}{\log 7} = \frac{\log 9}{\log 3} = \frac{\log 3^2}{\log 3} = \frac{2\log 3}{\log 3} = 2$

13. a) $A = \log 3B - \log C \rightarrow A = \log\frac{3B}{C} \rightarrow 10^A = \frac{3B}{C} \rightarrow B = \frac{C \cdot 10^A}{3}$

b) $1 + \log(AB) = \log C \rightarrow \log(AB) - \log C = -1 \rightarrow \log\left(\frac{AB}{C}\right) = -1 \rightarrow 10^{-1} = \frac{AB}{C} \rightarrow A = \frac{C}{10B}$

c) $3\log A + \log B = \log C \rightarrow \log A^3 + \log B = \log C \rightarrow \log A^3 \cdot B = \log C \rightarrow A^3 B = C \rightarrow A = \sqrt[3]{\frac{C}{B}}$

d) $\log A = \log B - C\log x \rightarrow \log A = \log B - \log x^c \rightarrow \log A = \log\frac{B}{x^c} \rightarrow x^c = \frac{B}{A} \rightarrow x = \left(\frac{B}{A}\right)^{\frac{1}{c}}$ or $\sqrt[c]{\frac{B}{A}}$

14. a) $\log_5(2x-1) + \log_5(x-2) = 1 \rightarrow \log_5(2x-1)(x-2) = 1 \rightarrow (2x-1)(x-2) = 5^1 \rightarrow$

$2x^2 - 5x + 2 = 5 \rightarrow 2x^2 - 5x - 3 = 0 \rightarrow (2x+1)(x-3) = 0 \rightarrow x = -\frac{1}{2}, 3$

Check: $(2x-1) \rightarrow 2\left(-\frac{1}{2}\right) - 1 = -2$, logarithm must be positive, therefore, reject $-\frac{1}{2}$

$(x-2) \rightarrow 3 - 2 = 1$, o.k. $(2x-1) \rightarrow 2 \cdot 3 - 1 = 5$, o.k. Therefore, $x = 3$

b) $\log_2(2-2x) + \log_2(1-x) = 5 \rightarrow \log_2(2-2x)(1-x) = 5 \rightarrow (2-2x)(1-x) = 2^5 \rightarrow$

$2 - 4x + 2x^2 = 32 \rightarrow 2x^2 - 4x - 30 = 0 \rightarrow x^2 - 2x - 15 = 0 \rightarrow (x-5)(x+3) = 0 \rightarrow x = 5, -3$

Check: $(1-x) \rightarrow 1 - 5 = -4$ logarithm must be positive, therefore, reject $x = 5$

$(1-x) \rightarrow 1-(-3) = 4$ o.k., $(2-2x) \rightarrow (2-2(-3))$ o.k. Therefore, $x = -3$

c) $\frac{1}{2} - \log_{16}(x-3) = \log_{16} x \rightarrow \log_{16} x + \log_{16}(x-3) = \frac{1}{2} \rightarrow \log_{16} x(x-3) = \frac{1}{2} \rightarrow x(x-3) = 16^{\frac{1}{2}} \rightarrow$

$x^2 - 3x - 4 = 0 \rightarrow (x-4)(x+1) = 0 \rightarrow x = 4, -1$ Check and reject -1 Therefore, $x = 4$

d) $\log_2(3x+1) + \log_2(x-1) = \log_2(10x+14) \rightarrow \log_2(3x+1)(x-1) = \log_2(10x+14) \rightarrow$

$(3x+1)(x-1) = 10x + 14 \rightarrow 3x^2 - 2x - 1 = 10x + 14 \rightarrow 3x^2 - 12x - 15 = 0 \rightarrow x^2 - 4x - 5 = 0 \rightarrow$

$(x-5)(x+1) = 0 \rightarrow x = -1, 5$ Check and reject $x = -1$ Therefore, $x = 5$

14. e) $\log_4(3x^2-5x-2)-\log_4(x-2)=1 \rightarrow \log_4\frac{(3x+1)(x-2)}{(x-2)}=1\rightarrow$

$\frac{(3x+1)(x-2)}{(x-2)}=4\rightarrow 3x+1=4\rightarrow 3x=3\rightarrow x=1$ Check and reject. Therefore, answer is ϕ

f) $\log x+\log(29-x)=2\rightarrow \log x(29-x)=2\rightarrow x(29-x)=10^2\rightarrow$
$-x^2+29x=100\rightarrow x^2-29x+100=0\rightarrow (x-25)(x-4)=0 \rightarrow x=4$ and 25
Check answers, both work. Therefore, $x=4$, 25

g) $\log_{25}(x-1)+\log_{25}(x+3)=\log_7\sqrt{7}\rightarrow \log_{25}(x-1)(x+3)=\frac{1}{2}\log_7 7=\frac{1}{2}\rightarrow$
$(x-1)(x+3)=25^{\frac{1}{2}}\rightarrow x^2+2x-3=5\rightarrow x^2+2x-8=0\rightarrow (x+4)(x-2)=0\rightarrow$
$x=-4$, 2 Check and reject -4 Therefore, $x=2$

h) $2\log(4-x)-\log 3 = \log(10-x) \rightarrow \log\frac{(4-x)^2}{3} = \log(10-x)\rightarrow$

$\frac{(4-x)^2}{3} = 10-x \rightarrow 16-8x+x^2 = 30-3x \rightarrow x^2-5x-14=0\rightarrow$
$(x-7)(x+2)=0\rightarrow x=-2,7$ Check and reject 7 Therefore, $x=-2$

i) $2\log_2(x+2)-\log_2(3x-2)=2\rightarrow \log_2\frac{(x+2)^2}{(3x-2)}=2\rightarrow \frac{(x+2)^2}{(3x-2)}=2^2\rightarrow x^2+4x+4=12x-8\rightarrow$

$x^2-8x+12=0\rightarrow (x-6)(x-2)=0,\ x=2,\ 6$ Check answers, both work Therefore, $x=2,\ 6$

j) $2\log_4 x+\log_4(x-2)-\log_4 2x=1\rightarrow \log_4\frac{x^2(x-2)}{2x}=1\rightarrow \frac{x(x-2)}{2}=4\rightarrow x^2-2x-8=0\rightarrow$
$(x-4)(x+2)=0,\ x=-2,\ 4$ Check and reject -2 Therefore, $x=4$

15. a) $\log\frac{x^3}{y^2} \rightarrow \log x^3-\log y^2 \rightarrow 3\log x-2\log y \rightarrow 3a-2b$

b) $\log_{16}81 = \frac{\log 81}{\log 16} = \frac{\log 3^4}{\log 2^4} = \frac{4\log 3}{4\log 2} = \log_2 3 = a$ Therefore, $\log_{16}81$ also equals "a"

c) $\log\frac{9}{5} = \log\frac{3^2}{25^{\frac{1}{2}}} = \log 3^2-\log 25^{\frac{1}{2}} = 2\log 3-\frac{1}{2}\log 25 = 2a-\frac{1}{2}b$

d) $\log\frac{25}{9} = \log 5^2-\log 3^2 = 2\log 5-2\log 3 = 2\log\frac{10}{2} - 2\log 3 = 2(\log 10-\log 2)-2\log 3 =$
$2(1-a)-2b = 2-2a-2b$

e) $\log_5 12=\frac{\log 12}{\log 5}=\frac{\log 2^2\cdot 3}{\log\frac{10}{2}}=\frac{2\log 2+\log 3}{\log 10-\log 2}=\frac{2a+b}{1-a}$

f) (i) $\log\frac{A}{B^2} = \log A-2\log B = 2-2(3) = 2-6 = -4$
(ii) $(\log AB)^2 = (\log A+\log B)^2 = (2+3)^2 = 25$

g) $\log AB=8\rightarrow \log A+\log B = 8$, if $\log B=-4$ then $\log A-4=8\rightarrow \log A=12$ Therefore, $A=10^{12}$

15. h) $\log_2\sqrt[3]{12.6} = \log_2\left(\frac{63}{5}\right)^{\frac{1}{3}} = \dfrac{\frac{1}{3}(\log 63 - \log 5)}{\log 2} = \dfrac{\frac{1}{3}(\log 3^2\cdot 7 - \log 5)}{\log\left(\frac{10}{5}\right)} =$

$\dfrac{\frac{1}{3}(2\log 3 + \log 7 - \log 5)}{\log 10 - \log 5} = \dfrac{\frac{1}{3}(2x + z - y)}{1-y} = \dfrac{\frac{2}{3}x + \frac{1}{3}z - \frac{1}{3}y}{1-y} = \dfrac{2x - y + z}{3 - 3y}$

i) $a = \log_8 3 \rightarrow a = \dfrac{\log 3}{\log 8}$, $b = \log_3 5 \rightarrow b = \dfrac{\log 5}{\log 3}$ Therefore, $ab = \dfrac{\log 3}{\log 8}\cdot\dfrac{\log 5}{\log 3} = \dfrac{\log 5}{\log 8}$

so $\log 5 = ab\log 8 = ab\log 2^3 = 3ab\log 2 = 3ab\log\frac{10}{5} = 3ab(\log 10 - \log 5) = 3ab(1 - \log 5) =$

$3ab - 3ab\log 5$, so, $3ab\log 5 + \log 5 = 3ab \rightarrow \log 5\,(3ab + 1) = 3ab \rightarrow \log 5 = \dfrac{3ab}{3ab+1}$

16. a) $2^{3x} = 5^{x-1} \rightarrow \log 2^{3x} = \log 5^{x-1} \rightarrow 3x\log 2 = (x-1)\log 5 \rightarrow$

$3x\log 2 = x\log 5 - \log 5 \rightarrow x\log 5 - 3x\log 2 = \log 5 \rightarrow x(\log 5 - 3\log 2) = \log 5 \rightarrow$

$x = \dfrac{\log 5}{\log 5 - 3\log 2}$ (acceptable answer) $\rightarrow x = \dfrac{\log 5}{\log\frac{5}{2^3}} = \log_{\frac{5}{8}} 5$ (better answer)

b) $7^{2x-1} = 17^x \rightarrow \log 7^{2x-1} = \log 17^x \rightarrow (2x-1)\log 7 = x\log 17 \rightarrow 2x\log 7 - \log 7 = x\log 17 \rightarrow$

$2x\log 7 - x\log 17 = \log 7 \rightarrow x(2\log 7 - \log 17) = \log 7 \rightarrow x = \dfrac{\log 7}{2\log 7 - \log 17}$ or $x = \log_{\frac{49}{17}} 7$

c) $3^{x-1} = 9\cdot 10^x \rightarrow \dfrac{3^{x-1}}{9} = 10^x \rightarrow \dfrac{3^{x-1}}{3^2} = 10^x \rightarrow 3^{x-3} = 10^x \rightarrow \log 3^{x-3} = \log 10^x \rightarrow$

$(x-3)\log 3 = x\log 10 \rightarrow x\log 3 - 3\log 3 = x \rightarrow x\log 3 - x = 3\log 3 \rightarrow$

$x(\log 3 - 1) = \log 3^3 \rightarrow x = \dfrac{\log 27}{\log 3 - 1}$ or $\dfrac{\log 27}{\log 3 - \log 10}$ or $\dfrac{\log 27}{\log\frac{3}{10}}$ or $\log_{\frac{3}{10}} 27$

d) $7^{x-1} = 2\cdot 5^{1-2x} \rightarrow \log 7^{x-1} = \log 2\cdot 5^{1-2x} \rightarrow (x-1)\log 7 = \log 2 + (1-2x)\log 5 \rightarrow$

$x\log 7 - \log 7 = \log 2 + \log 5 - 2x\log 5 \rightarrow x\log 7 + 2x\log 5 = \log 2 + \log 5 + \log 7 \rightarrow$

$x(\log 7 + 2\log 5) = \log 2 + \log 5 + \log 7 \rightarrow x = \dfrac{\log 2 + \log 5 + \log 7}{\log 7 + 2\log 5}$ or $\dfrac{\log 70}{\log 175}$ or $\log_{175} 70$

17. a) $\log_2(\log_8 x) = -1 \rightarrow \log_8 x = 2^{-1} \rightarrow \log_8 x = \dfrac{1}{2} \rightarrow x = 8^{\frac{1}{2}} = (2^3)^{\frac{1}{2}} \rightarrow$

$x = 2^{\frac{3}{2}}$ or $2\sqrt{2}$

b) $\log_2(\log_x(\log_3 27)) = -1 \rightarrow \log_x(\log_3 27) = 2^{-1} \rightarrow \log_x(\log_3 3^3) = \dfrac{1}{2} \rightarrow$

$\log_x 3(\log_3 3) = \dfrac{1}{2} \rightarrow \log_x 3 = \dfrac{1}{2} \rightarrow x^{\frac{1}{2}} = 3 \rightarrow x = 3^2 = 9$

c) $\log_{\frac{1}{2}}(\log_4(\log_2 x)) = 1 \rightarrow \log_4(\log_2 x) = \dfrac{1}{2} \rightarrow \log_2 x = 4^{\frac{1}{2}} = 2 \rightarrow x = 2^2 = 4$

18. a) $\log x = \frac{2}{3}\log 27 + 2\log 2 - \log 3 = \log\frac{27^{\frac{2}{3}}\cdot 2^2}{3} \rightarrow x = \frac{27^{\frac{2}{3}}\cdot 2^2}{3} = \frac{9\cdot 4}{3} = 12$

b) $\log y = \log 2 + 3\log_{\sqrt{10}} x - \log 2z = \log 2 + \frac{\log x^3}{\log\sqrt{10}} - \log 2z \rightarrow$

$\log y = \log 2 + \frac{\log x^3}{\frac{1}{2}\log 10} - \log 2z = \log 2 + 2\log x^3 - \log 2z \rightarrow$

$\log y = \log 2 + \log x^6 - \log 2z = \log\frac{2x^6}{2z} \rightarrow y = \frac{x^6}{z}$

c) $2\log x = -\log a + 3\log b + 4\log\frac{1}{c} \rightarrow \log x^2 = \log a^{-1} + \log b^3 + \log\left(\frac{1}{c}\right)^4 \rightarrow$

$\log x^2 = \log a^{-1}\cdot b^3\cdot\left(\frac{1}{c}\right)^4 \rightarrow x^2 = \frac{b^3}{ac^4} \rightarrow x = \sqrt{\frac{b^3}{a\cdot c^4}} \rightarrow x = \sqrt{\frac{b^2\cdot b\cdot a}{a^2\cdot c^4}} = \frac{b\sqrt{ab}}{ac^2}$

19. a) $x = \frac{a^2}{b^3c^{\frac{1}{2}}} \rightarrow \log x = \log\frac{a^2}{b^3c^{\frac{1}{2}}} \rightarrow \log x = 2\log a - 3\log b - \frac{1}{2}\log c$

b) $x = \frac{a^{-2}b^3}{c^{-\frac{1}{2}}} \rightarrow x = \frac{b^3c^{\frac{1}{2}}}{a^2} \rightarrow \log x = \log\frac{b^3c^{\frac{1}{2}}}{a^2} \rightarrow \log x = 3\log b + \frac{1}{2}\log c - 2\log a$

c) $x = \frac{\sqrt[3]{a^2}\cdot b^{-\frac{2}{5}}}{c^{\frac{1}{2}}} \rightarrow \log x = \log\frac{\sqrt[3]{a^2}\cdot b^{-\frac{2}{5}}}{c^{\frac{1}{2}}} \rightarrow \log x = \frac{2}{3}\log a - \frac{2}{5}\log b - \frac{1}{2}\log c$

20. a) $\log_2 16^{2x+1} = 8 \rightarrow 2^8 = 16^{2x+1} \rightarrow 2^8 = 2^{4(2x+1)} \rightarrow 2^8 = 2^{8x+4} \rightarrow 8x+4 = 8 \rightarrow 8x = 4 \rightarrow x = \frac{1}{2}$

b) $\log_{16} x + \log_4 x + \log_2 x = 7 \rightarrow \frac{\log x}{\log 16} + \frac{\log x}{\log 4} + \frac{\log x}{\log 2} = 7 \rightarrow \frac{\log x}{\log 2^4} + \frac{\log x}{\log 2^2} + \frac{\log x}{\log 2} = 7 \rightarrow$

$\frac{\log x}{4\log 2} + \frac{\log x}{2\log 2} + \frac{\log x}{\log 2} = 7 \rightarrow \frac{1}{4}\log_2 x + \frac{1}{2}\log_2 x + \log_2 x = 7 \rightarrow \frac{7}{4}\log_2 x = 7 \rightarrow \log_2 x = 4 \rightarrow x = 2^4 = 16$

c) $\log_9 x + 3\log_3 x = 7 \rightarrow \frac{\log_3 x}{\log_3 9} + 3\log_3 x = 7 \rightarrow \frac{\log_3 x}{\log_3 3^2} + 3\log_3 x = 7 \rightarrow$

$\frac{1}{2}\log_3 x + 3\log_3 x = 7 \rightarrow \frac{7}{2}\log_3 x = 7 \rightarrow \log_3 x = 7\cdot\frac{2}{7} = 2 \rightarrow x = 3^2 = 9$

d) $2\log_4 x - 3\log_x 4 = 5 \rightarrow 2\log_4 x - \frac{3}{\log_4 x} = 5 \rightarrow 2(\log_4 x)^2 - 3 = 5\log_4 x \rightarrow$

$2(\log_4 x)^2 - 5\log_4 x - 3 = 0 \rightarrow (2\log_4 x + 1)(\log_4 x - 3) = 0 \rightarrow$

$\log_4 x = -\frac{1}{2} \rightarrow x = 4^{-\frac{1}{2}} = \frac{1}{2}$ and $\log_4 x = 3 \rightarrow x = 4^3 = 64$ Therefore, $x = \frac{1}{2}$ or 64

e) $(\log_4 a)(\log_a 2a)(\log_{2a} x) = \log_a a^3 \rightarrow \frac{\log a}{\log 4}\cdot\frac{\log 2a}{\log a}\cdot\frac{\log x}{\log 2a} = 3 \rightarrow$

$\frac{\log x}{\log 4} = 3 \rightarrow \log_4 x = 3 \rightarrow x = 4^3 \rightarrow x = 64$

1. a) $A = A_0(x)^{\frac{t}{T}} \rightarrow 10\,000 = 40\,000(0.85)^{\frac{t}{1}} \rightarrow 0.25 = 0.85^t \rightarrow \log_{0.85} 0.25 = t$

$t = \dfrac{\log\ 0.25}{\log\ 0.85} = 8.53$ It takes 8.53 years to depreciate to \$10 000.

b) $A = P\left(1+\dfrac{r}{n}\right)^{nt} \rightarrow 1\,000\,000 = 10\,000\left(1+\dfrac{0.12}{4}\right)^{4t} \rightarrow 100 = 1.03^{4t} \rightarrow$

$\log_{1.03} 100 = 4t \rightarrow t = \dfrac{\log 100}{4\log 1.03} = 38.95$ years.

c) $A = P\left(1+\dfrac{r}{n}\right)^{nt} \rightarrow 3P = P\left(1+\dfrac{r}{2}\right)^{2\cdot 15} \rightarrow 3 = \left(1+\dfrac{r}{2}\right)^{30} \rightarrow 1+\dfrac{r}{2} = 3^{\frac{1}{30}} \rightarrow$

$\dfrac{r}{2} = 3^{\frac{1}{30}} - 1 \rightarrow r = 2\,(3^{\frac{1}{30}} - 1) = 7.46\%$

d) Method 1: $A = A_0(x)^{\frac{t}{T}} \rightarrow 0.8 = 1\left(\dfrac{1}{2}\right)^{\frac{30}{T}}$ (if 20% lost , 80% remains) $\rightarrow \log_{\frac{1}{2}} 0.8 = \dfrac{30}{T} \rightarrow$

$T = \dfrac{30}{\log_{\frac{1}{2}} 0.8} = \dfrac{30}{\dfrac{\log 0.8}{\log \frac{1}{2}}} = \dfrac{30\log\frac{1}{2}}{\log 0.8} = 93.2$ hours.

Method 2: $A = A_0 e^{kt} \rightarrow 0.8 = 1\cdot e^{k\cdot 30} \rightarrow \ln 0.8 = 30\,k \rightarrow k = \dfrac{\ln\ 0.8}{30}$

$A = A_0 e^{\left(\frac{\ln 0.8}{30}\right)t} \rightarrow \dfrac{1}{2} = 1\cdot e^{\left(\frac{\ln 0.8}{30}\right)t} \rightarrow \ln 0.5 = \left(\dfrac{\ln\ 0.8}{30}\right)t \rightarrow$

$t = \dfrac{30\ln 0.5}{\ln 0.8} = 93.2$ hours

e) Remember, the **smaller** the pH values of an acidic solution the **stronger** the acidity. The **larger** the pH value of an alkaline solution the **stronger** the alkalinity.

(*i*) $4.8 - 2.1 = 2.7$, then $10^{2.7} = 501$, therefore, lemon juice is 501 times more acidic than black coffee.

(*ii*) $10^x = 75 \rightarrow x = \log 75 \rightarrow x = 1.9$, therefore, $4.2 + 1.9 = 6.1$ is the pH of milk.

f) $A = A_0(x)^{\frac{t}{T}} \rightarrow 400\,000\,(1.02)^t = 300\,000\,(1.03)^t \rightarrow 1.02^t = \dfrac{3}{4}(1.03)^t \rightarrow$

$\log 1.02^t = \log\dfrac{3}{4}(1.03)^t = \log\dfrac{3}{4} + \log 1.03^t \rightarrow$

$t\log 1.02 = \log\dfrac{3}{4} + t\log 1.03 \rightarrow t\,(\log 1.02 - \log 1.03) = \log\dfrac{3}{4} \rightarrow$

$t = \dfrac{\log\frac{3}{4}}{\log\frac{1.02}{1.03}} \approx 29.5$ Surrey catches up in population to Vancouver in 29.5 years.

21. g) $A = P\left(1+\frac{r}{n}\right)^{nt} \rightarrow 3 = 1\left(1+\frac{0.08}{365}\right)^{365t} \rightarrow 365t = \frac{\log 3}{\log\left(1+\frac{0.08}{365}\right)} \rightarrow t = \frac{\log 3}{365\log\left(1+\frac{0.08}{365}\right)} = 13.7$ *years*

h) $C = 8e^{0.3t} \rightarrow 100 = 8e^{0.3t} \rightarrow 12.5 = e^{0.3t} \rightarrow 0.3t = \ln 12.5 \rightarrow t = \frac{\ln 12.5}{0.3} = 8.42°\text{C}$

$t = 8.42$ degrees Celsius

i) $P(t) = 4\,000\,000\ e^{0.012t} \rightarrow 6\,400\,000 = 4\,000\,000\ e^{0.012t} \rightarrow 1.6 = e^{0.012t} \rightarrow$

$\ln 1.6 = 0.012t \rightarrow t = \frac{\ln 1.6}{0.012} = 39.2$

Therefore, in year 2039 the population will reach 6 400 000.

j) Method 1: $A = A_0(x)^{\frac{t}{T}} \rightarrow 100\,000 = 1\,200(2)^{\frac{t}{4}} \rightarrow \frac{250}{3} = 2^{\frac{t}{4}} \rightarrow$

$\log_2 \frac{250}{3} = \frac{t}{4} \rightarrow t = \frac{4\log\frac{250}{3}}{\log 2} = 25.5$ days

Method 2: $A = A_0e^{kt} \rightarrow 2 = 1\cdot e^{k\cdot 4} \rightarrow \ln 2 = 4k \rightarrow k = \frac{\ln 2}{4} \rightarrow A = A_0e^{\left(\frac{\ln 2}{4}\right)t} \rightarrow$

$100\,000 = 1200\,e^{\left(\frac{\ln 2}{4}\right)t} \rightarrow \frac{250}{3} = e^{\left(\frac{\ln 2}{4}\right)t} \rightarrow \ln\left(\frac{250}{3}\right) = \left(\frac{\ln 2}{4}\right)t \rightarrow$

$t = \frac{4\ln\left(\frac{250}{3}\right)}{\ln 2} = 25.5$ days

k) $A = A_0e^{kt} \rightarrow \frac{1}{2} = 1\cdot e^{k\cdot 5570} \rightarrow \ln 0.5 = 5\,570\,k \rightarrow k = \frac{\ln 0.5}{5570}$

$A = A_0e^{\left(\frac{\ln 0.5}{5570}\right)t} = 500\,e^{\left(\frac{\ln 0.5}{5570}\right)2500} = 366.3$ grams

22. a) In step 6, you are dividing by $\log\frac{1}{2}$ which is a negative number, therefore, the direction of the inequality **must be changed.**

b) When you multiply by $\log\frac{1}{2}$ in step 2, the value of two **positive** numbers is changed to two **negative** numbers, without changing the direction of the inequality.

23. Let $q = 2^{30402457} - 1$ with $2 = 10^x \rightarrow x = \log 2 \rightarrow 2 = 10^{\log 2}$

Substitute $q = \left(10^{\log 2}\right)^{30402457} - 1$

$= 10^{9152051.5} - 1$

But 10^n has $n+1$ digits so q has 9 152 052 digits.

(By the way, printing this number would take about 1 500 *pages!*)

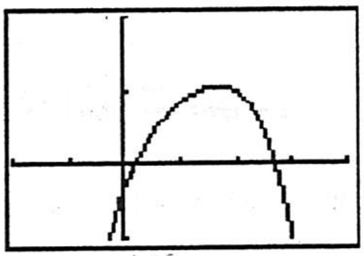
24. a)

$Y_1 = 2^{1-x} - \log x$

x [−1, 6] y [−1, 4]

The solution is the crossing of the x-axis.
Therefore, the solution is $x = 2.40$

b)

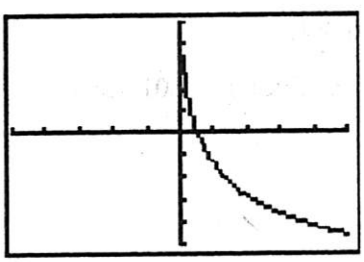


$Y_1 = 2^x - 3^{x-1} - 6^{-x}$

x [−2, 4] y [−1, 2]

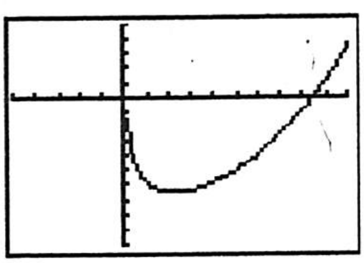
The solution is the crossing of the x-axis.
Therefore, the solution is $x = 0.18, 2.71$

c)



$Y_1 = 2^x - \log x - 3$

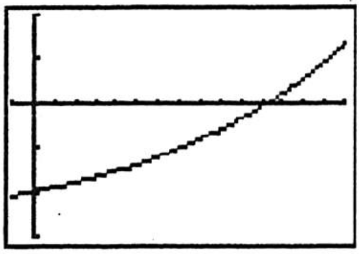
x [−0.1, 0.1] y [−1, 1]



x [−1, 2] y [−2, 1]

The zero from the first window is 0.010, and the zero from the second is 1.69.
Therefore, the solutions are: $x = 0.010, 1.69$

24. d)



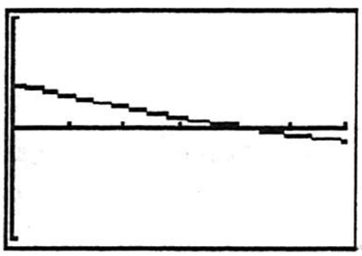
$Y_1 = 1.05^{2x} - 3$

x [−1, 15] y [−3, 2]

Formula for compound interest is $A = P\left(1+\frac{r}{n}\right)^{nt} \rightarrow 3 = 1\left(1+\frac{0.1}{2}\right)^{2t} \rightarrow 3 = 1.05^{2t}$

Therefore, graph Y_1 and find the zeros of the equation, which gives $t = 11.26$ years.

e)



$Y_1 = \left(\frac{1}{2}\right)^{\frac{x}{5700}} - 0.6$

x [0, 6000] y [−1, 1]

Formula for growth and decade is $A = A_0(x)^{\frac{t}{T}} \rightarrow 60 = 100\left(\frac{1}{2}\right)^{\frac{t}{5700}}$.

Therefore, graph Y_1 and find the zeros of the equation, which gives $t = 4201$ years.

Logarithms – Multiple-choice Solutions

ANSWERS

1. a	8. d	15. b	22. b	29. d	36. b	43. a	50. b	57. a	64. d
2. d	9. c	16. a	23. a	30. a	37. d	44. b	51. b	58. b	65. a
3. c	10. c	17. b	24. c	31. a	38. a	45. c	52. d	59. d	
4. b	11. a	18. c	25. c	32. d	39. c	46. b	53. d	60. d	
5. b	12. b	19. c	26. c	33. d	40. b	47. d	54. a	61. d	
6. d	13. a	20. b	27. b	34. d	41. c	48. c	55. b	62. a	
7. d	14. a	21. c	28. c	35. c	42. b	49. c	56. a	63. a	

LIST OF **WRONGS** AND **RIGHTS**

Probably the main overall rule to keep upper most in your mind when working with logarithms is as follows: **Do not** make up your own rules!

Wrong	**Right**
1. $(\log a)^n = n \log a$	1. $(\log a)^n$ cannot be simplified
2. $\log x \cdot \log y = \log xy$	2. $\log x \cdot \log y$ cannot be simplified
3. $\log(x + y) = \log x + \log y$	3. $\log(x + y)$ cannot be simplified
4. $\log x \cdot \log x = \log x^2$	4. $\log x \cdot \log x = (\log x)^2$
5. $\log_y x = \log x - \log y$	5. $\log_y x = \dfrac{\log_b x}{\log_b y}$
6. $\dfrac{\log x}{\log y} = \log x - \log y$	6. $\dfrac{\log x}{\log y} = \log_y x$
7. $\log \dfrac{x}{2} = \dfrac{\log x}{2}$	7. $\log \dfrac{x}{2} = \log x - \log 2$
8. $\dfrac{\log 10}{\log 5} = \dfrac{10}{5} = 2$ or $\dfrac{\log 10}{\log 5} = \log 2$	8. $\dfrac{\log 10}{\log 5} = \log_5 10 \approx 1.43$
9. $\dfrac{\log_2 5}{5} = \log_2$	9. $\dfrac{\log_2 5}{5}$ cannot be simplified
10. $b^x + b^y = b^{x+y}$	10. $b^x + b^y$ cannot be simplified