Oral Exam – Practice questions

1 point questions

1. An angle, in radians, that is co-terminal with 30° is $\frac{13\pi}{6}$ for example

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17. Given that $f(\theta) = \cos(n\theta)$ has the same period as the graph of $g(\theta) = \tan\theta$, the value of n is 2.

$p = \pi \Rightarrow n = 2$

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16. An angle that is co-terminal with an angle of $\frac{-11\pi}{4}$, in standard position, is $\frac{5\pi}{4}$ for example

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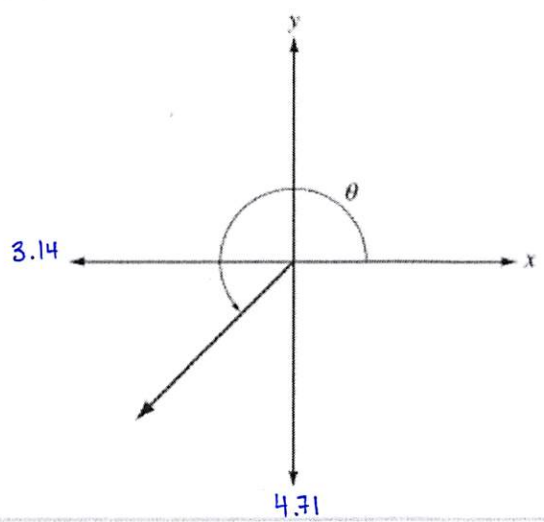
2 point questions

Use the following information to answer the next question.

An angle, θ, in standard position, is shown below.

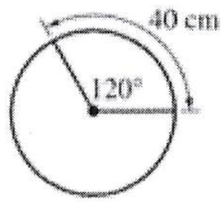


2. The best estimate of the rotation angle θ is

A. 1.25 radians
B. 3.12 radians
*C. 4.01 radians
D. 5.38 radians

Use the following information to answer the next question.

Mary is given the diagram below, showing an angle rotation of 120°. The arc length of the sector is 40 cm.



$a = r\theta$ $\theta = \frac{2\pi}{3}$

$40 = r \times \frac{2\pi}{3}$

$\frac{120}{2\pi} = r$

3. What is the radius of the circle? $r = \frac{60}{\pi} \approx 19$ cm

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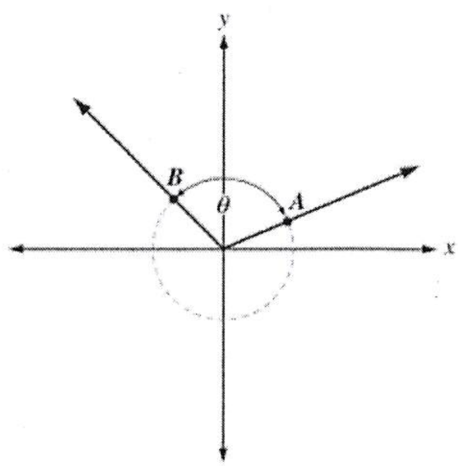
Use the following information to answer the next question.

Point $A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ and Point $B\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ lie on the terminal arm of two different angles in standard position on the circle. The angle, θ, where $0 < \theta < \pi$, can be expressed in the form $\frac{m\pi}{n}$.



$\theta_A = \frac{\pi}{6}$

$\theta_B = \frac{3\pi}{4}$

$\theta = \frac{3\pi}{4} - \frac{\pi}{6} = \frac{9\pi}{12} - \frac{2\pi}{12} = \frac{7\pi}{12}$

6. The values of m and n are, respectively, 7 and 12.

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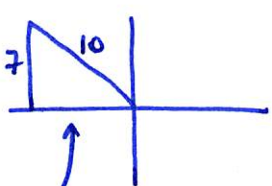
8. The terminal arm of θ, when drawn in standard position, contains the point $P(x, y)$ where P is on the unit circle. If $\sin\theta = \frac{7}{10}$ and $\tan\theta < 0$, then what is the value of x?



quad II

$x = \cos\theta$

$= \boxed{-\frac{\sqrt{51}}{10}}$

Pyth: $\sqrt{51}$

Use the following information to answer the next question.

If the point $P(0.2, k)$ lies on a circle with a centre at the origin and a radius of 1, then the exact value of k can be expressed as $\pm\sqrt{b}$.

$0.2^2 + k^2 = 1$

$k^2 = 1 - 0.04$

$k^2 = 0.96$

7. The value of b, to the nearest hundredth, is 0.96.

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9. On a unit circle, Point $P\left(-\frac{5}{13}, \frac{12}{13}\right)$ lies on the terminal arm of angle θ in standard position.

x y

What are the exact values of the 6 trigonometric ratios for angle θ?

$x^2 + y^2 = 1 \Rightarrow$ unit circle

$\cos\theta = -\frac{5}{13}$ $\sin\theta = \frac{12}{13}$ $\tan\theta = -\frac{12}{5}$ $\sec\theta = -\frac{13}{5}$ $\csc\theta = \frac{13}{12}$ $\cot\theta = -\frac{5}{12}$

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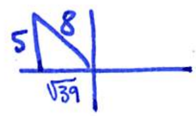
10. Given that $\csc\theta = \frac{8}{5}$, where $\frac{\pi}{2} < \theta < \pi$, determine the **exact** value of $\tan\theta$.



$\sin\theta = \frac{5}{8}$ $\tan\theta = -\frac{5}{\sqrt{39}}$

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11. The **exact** value of $\sin\left(-\frac{\pi}{6}\right) + \cos\left(\frac{7\pi}{4}\right)$ is

$-\frac{1}{2} + \frac{\sqrt{2}}{2} = \frac{-1+\sqrt{2}}{2}$

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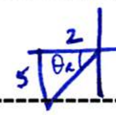
12. If $\tan\theta = \frac{5}{2}$, where $0 \le \theta < 2\pi$, then the largest positive value of θ, to the nearest tenth, is 4.3 rad.



$\theta = \pi + \theta_R$ $\theta_R = \tan^{-1}\left(\frac{5}{2}\right)$

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13. Determine the exact values of the following ratios:

$\tan\left(\frac{\pi}{2}\right)$ = undef $\cot\left(\frac{3\pi}{2}\right) = 0$ $\sin\pi = 0$ $\csc(2\pi)$ = undef

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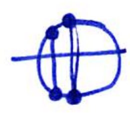
21. The values of θ, where $180° \le \theta < 360°$, in the equation $2\cos^2\theta + \cos\theta = 0$, are

240° ; 270°

$\cos\theta(2\cos\theta + 1) = 0$ → $\cos\theta = 0$ → $\cos\theta = -\frac{1}{2}$



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23. Determine a general solution of $\cot^2\theta - 1 = 0$, expressed in radians.

$\cot^2\theta = 1$

$\cot\theta = \pm 1$

$\tan\theta = \pm 1$

$\boxed{\theta = \frac{\pi}{4} + \frac{\pi}{2}n, n \in \mathbb{Z}}$

Use the following information to answer the next question.

For the angles $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, and $\frac{11\pi}{6}$, the following statements are given.

Statement 1 These angles in degrees are, respectively, 30°, 150°, 210°, and ~~300~~° 330°. ✗

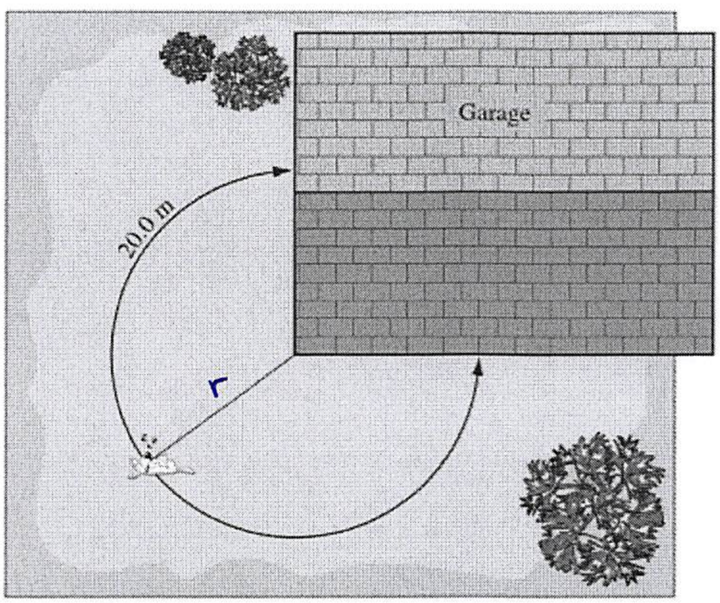
Statement 2 They are all part of the solution set $\theta = \frac{\pi}{6} + 2n\pi,\ n \in I$. ✗

Statement 3 The values of $\sin\left(\frac{7\pi}{6}\right)$ and $\cos\left(\frac{5\pi}{6}\right)$ are both negative. ✓

14. The statement that is true from the list above is number 3.

Use the following information to answer numerical-response question 8.

A dog is tied to the corner of a rectangular garage. He is given enough leash to run along a 20.0 m circular path, completing $\frac{3}{4}$ of a circle, as shown in the diagram below.



$a = r\theta$

$20 = r \times \frac{3\pi}{2}$

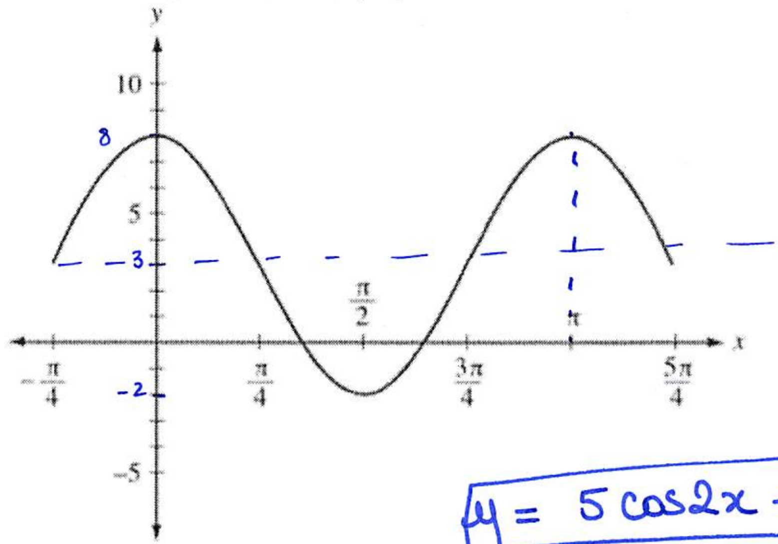
$\frac{40}{3\pi} = r$

Numerical Response

8. The length of the dog's leash, to the nearest tenth of a metre, is 4.2 m.

15. What is an equation of this graph?

$\boxed{\cos}$



$a = 5$

$b = \frac{2\pi}{\pi} = 2$

$h = \phi$

$k = 3$

$\boxed{y = 5\cos 2x + 3}$

$2 - \sqrt{3}\sec\theta = 0$ $\quad \sec\theta + 3 = 0$

$\sec\theta = \frac{2}{\sqrt{3}}$ $\quad \cos\theta = \frac{\sqrt{3}}{2}$ $\quad \pi/6 \quad \frac{11\pi}{6}$

$\sec\theta = -3$ $\quad \cos\theta = -\frac{1}{3}$ $\quad \theta_R \approx 70.5°$

25. Solve $(2 - \sqrt{3}sec\theta)(sec\theta + 3) = 0$ *where* $-180° \leq \theta \leq 0°$

$\boxed{\theta = -30° \quad \theta \approx -109°}$

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29. Simplify the following expressions as much as possible.

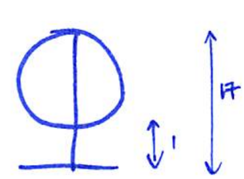
1 $\sin x - \frac{\tan x}{\sec x} = \sin x - \frac{\sin x}{\cos x} \times \cos x = \boxed{0}$

2 $\cot^2 x - \csc^2 x = \frac{\cos^2 x}{\sin^2 x} - \frac{1}{\sin^2 x} = \frac{\cos^2 x - 1}{\sin^2 x} = \frac{-\sin^2 x}{\sin^2 x} = \boxed{-1}$

3 $\frac{1}{7}\cos^2 x + \frac{1}{7}\sin^2 x = \frac{1}{7}(\underbrace{\cos^2 x + \sin^2 x}_{1}) = \boxed{\frac{1}{7}}$

Use the following information to answer the next question.

> The height of a point on a Ferris wheel, h, in metres above the ground, as a function of time, t, in seconds can be represented by a sinusoidal function. The maximum height of the Ferris wheel above the ground is 17 m and the minimum height is 1 m. It takes the Ferris wheel 60 seconds to complete two full rotations.



p=30 s

20. Assuming that the particular point starts at the minimum height above the ground, write an equation for the height of this point on the Ferris wheel, h, as a function of time, t, in the form $h = a\cos[b(t-c)] + d$.

$-8 \quad \frac{\pi}{15} \quad 0 \quad 9$

$\boxed{h = -8\cos\left(\frac{\pi}{15}t\right) + 9}$

30. What is the **exact** value of $\tan 75°$? $= \tan(30° + 45°) = \dfrac{\tan 30 + \tan 45}{1 - \tan 30 \cdot \tan 45} = \dfrac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}} = \dfrac{\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$

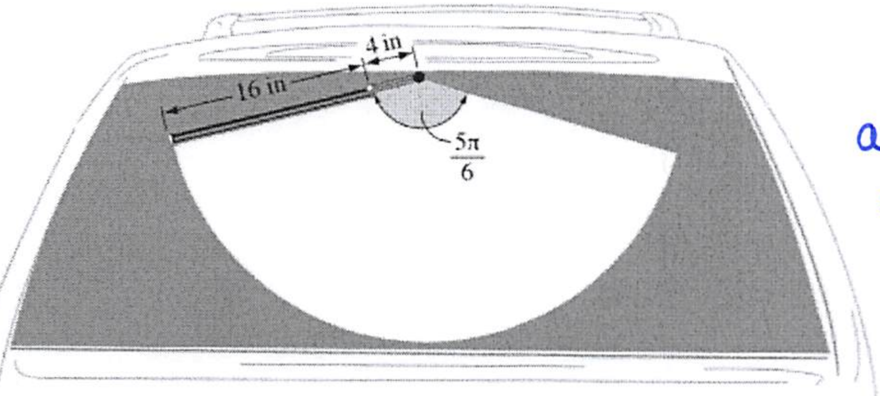
$= \dfrac{1+\sqrt{3}}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \dfrac{1+2\sqrt{3}+3}{3-1}$

$= \boxed{2+\sqrt{3}}$

Use the following information to answer the next question.

The wiper on the rear window of a particular car moves through an angle of $\frac{5\pi}{6}$ rad in a single sweep and clears a region that is 16 in wide, as shown in the diagram below.



$a = r\theta$

$= 16 \times \dfrac{5\pi}{6} = \dfrac{40\pi}{3}$

7. The total perimeter of the cleared region, to the nearest inch, is $P = a + 2 \times 16 = \dfrac{40\pi}{3} + 32 \approx \boxed{74 \text{ in}}$

Use the following information to answer the next question.

Point $A(x, y)$ is the intersection point of the unit circle and the terminal arm of 145° in standard position. $145° \longrightarrow (\cos 145 ; \sin 145)$

8. The coordinates of Point A, to the nearest hundredth, are $(-0{,}82 ; 0{,}57)$

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7. An expression that is equivalent to $\log\left(\dfrac{2\sin x}{\sin(2x)}\right)$, where $0° < x < 90°$, is

$= \log\left(\dfrac{2\sin x}{2\sin x . \cos x}\right) = \log\left(\dfrac{1}{\cos x}\right) = -\log(\cos x)$

A. $\log 1$
B. $\log(\cos x)$
C. $-\log(\sin x)$
D. $-\log(\cos x)$

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19. *If* $\csc\theta = \dfrac{2}{\sqrt{3}}$, *where* $0 \le \theta < 2\pi$, *then* θ *lies in Quadrant* ___*i*___, *and* $\boxed{\text{I or II}}$

$\cot\theta$ *is equal to* ___*ii*___. $\boxed{\pm\frac{1}{\sqrt{3}}}$

$\sin\theta = \dfrac{\sqrt{3}}{2}$

$p = 120°$ $3x-20=60$ $3x-20=300$

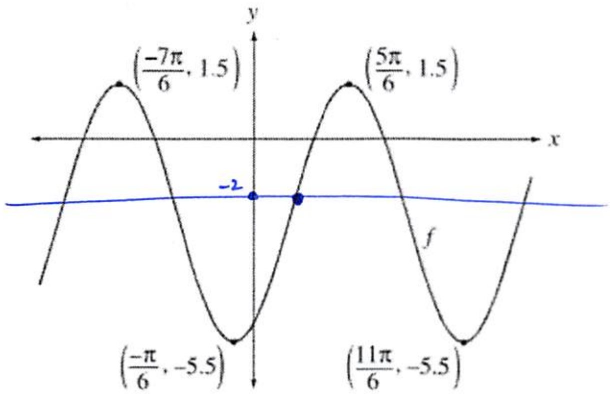
3. Solve $2\cos(3x - 20°) = 1,\ where\ 0° \le x \le 90°$ $3x = 80$ $3x = 320$

$\cos(3x-20°) = \frac{1}{2}$ $x = \frac{80}{3} + 120n$ $x = \frac{320}{3} + 120n$

$300°$ $\boxed{sol: \{\frac{80}{3}\}}$

Use the following information to answer question 20.

The partial graph of $f(x) = a\sin[b(x-c)] + d$, where $a > 0$ and x is in radians, is shown below. Two of its maximum points and two of its minimum points are labelled.



$a = 3.5$

$b = 1$

$h = \frac{\pi}{3}$

$k = -2$

$\boxed{f(x) = 3.5\sin\left(x - \frac{\pi}{3}\right) - 2}$

2π

20. The minimum positive value of c, to the nearest hundredth of a radian, is $\frac{\pi}{3} \approx \boxed{1.05}$

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17. Solve: $2\sin^2 x + 5\sin x - 3 = 0$ $\otimes -6$, $\oplus 5$ } $-6, \cancel{6}, 1$

$3\sin^2 x + \cos^2 x + 5\sin x - 4 = 0$, where $0 \le x < 2\pi$. $\sin x = \frac{1}{2}$ $\sin x = -3$ impossible $\boxed{sol: \{\frac{\pi}{6}; \frac{5\pi}{6}\}}$

$3\sin^2 x + 1 - \sin^2 x + 5\sin x - 4 = 0$

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23. The non-permissible values of θ for the identity $\frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$ are $\cos^2\theta \ne 1$ $\sin\theta \ne 0$

$\boxed{\theta \ne \pi n, n \in \mathbb{Z}}$ $\cos\theta \ne \pm 1$

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24. If the identity $\frac{\sin x}{1-\cos x} = \frac{1+\cos x}{\sin x}$ is verified using $x = \frac{2\pi}{3}$, then the **exact** value of each side is

$\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ L.S: $\frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{\sqrt{3}}{2} \times \frac{2}{3} = \boxed{\frac{\sqrt{3}}{3}}$

$\cos\frac{2\pi}{3} = -\frac{1}{2}$

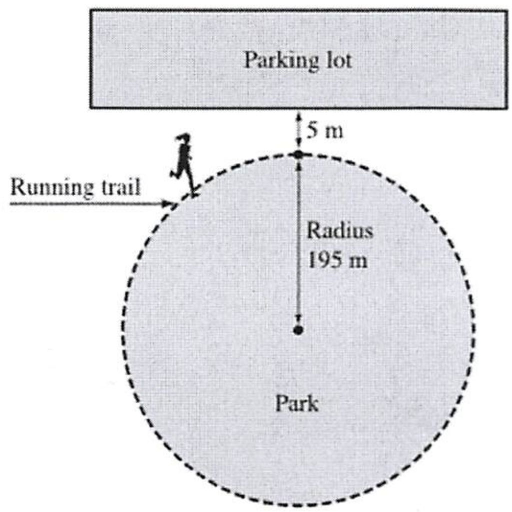
Use the following information to answer question 21.

A local park with a radius of 195 m has a circular running trail surrounding it, as shown below. The shortest distance from the running trail to the parking lot is 5 m. At a constant speed, Ellie can complete 4 full laps around the park in 32 minutes. Ellie's distance from the parking lot as she runs, s, in metres, as a function time, t, in minutes, can be represented by the function

$-195 \quad \frac{\pi}{4} \quad 0 \quad 200$

$$s(t) = a\cos\left[b(t-c)\right] + d.$$

1 lap: 8 min = p



21. The values of $\boldsymbol{b}$ and $\boldsymbol{d}$ in the function are, respectively, $\frac{\pi}{4}$ & 200

3 points questions

1. a)

   7 3

   For the function $y = a\cos\theta + d$, the range is $[-4, 10]$, the values of $\boldsymbol{a}$ and $\boldsymbol{d}$ are, respectively, __7__, and __3__.

   b)

   A function is represented by the equation $y = \sin(3x + \pi) + 7$. *The value of the phase shift is __i__ and the period of the corresponding graph is __ii__ .*

   $\frac{\pi}{3}$ $\frac{2\pi}{3}$

Use the following information to answer the next question.

For the graph of the function $f(x) = -3\sin\left[2(x-5)\right] + d$, the following statements were made.

Statement 1 The amplitude is 3. ✓

Statement 2 The maximum value is $(d-3)$. ✓

Statement 3 The period is 2π. ✗

Statement 4 When compared to the graph of $g(x) = -3\sin(2x) + d$, the graph of $y = f(x)$ has been horizontally translated 5 units to the right. ✓

Statement 5 If $d > 3$, then the graph of $y = f(x)$ will have no x-intercepts. ✓

19. The true statements are 1, 2, 4 & 5

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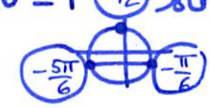
24. Determine the solution set for the equation $2\cos^2 x + \sin x - 1 = 0$, where $-\pi \le x \le \pi$.

$2(1-\sin^2 x) + \sin x - 1 = 0$ $\quad 2\sin^2 x - \sin x - 1 = 0$

$\sin\theta = 1$ $\frac{\pi}{2}$ $\quad \sin\theta = -\frac{1}{2}$



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3. Determine the restrictions on the variable and simplify the expression: $\dfrac{\cot x + \csc x}{\sec x + 1}$.

$\boxed{x \ne \pi n, n \in \mathbb{Z}}$

$\sin x \ne 0$ $\quad \sec x \ne -1$

$\cos x \ne -1$

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29. Determine the restrictions on the variable and simplify the expression:

$$\frac{\cos(2x) + 2\sin^2 x}{1 + \tan^2 x}, = \frac{1 - 2\sin^2\theta + 2\sin^2\theta}{\sec^2\theta} = \boxed{\cos^2\theta}$$

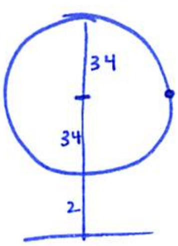
no restrictions.

30.

The London Eye is an observation wheel with a radius of 68 m. Rides consist of one complete revolution, which takes 30 minutes, and begin on a platform 2 m above the ground.

The height, $h(t)$, in metres, of a rider above the ground as a function of time, t, in minutes, can be expressed as $h(t) = a\sin\left[b(t-c)\right] + d$.

$a = 34$ $\quad b = \frac{\pi}{15}$ $\quad c = 7.5$ $\quad d = 36$



$p = 30$