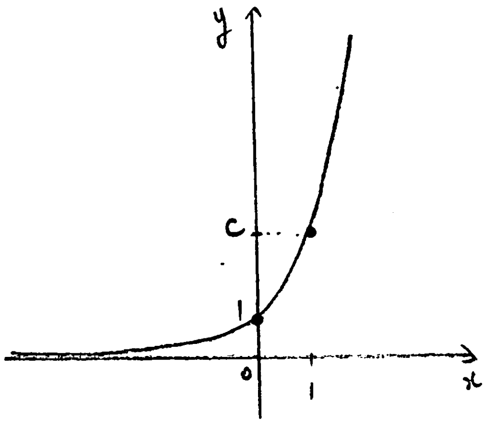
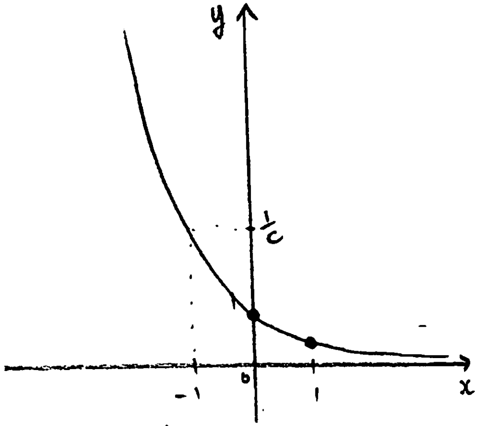
EXPONENTIAL FUNCTIONS $\boxed{y = c^x}$

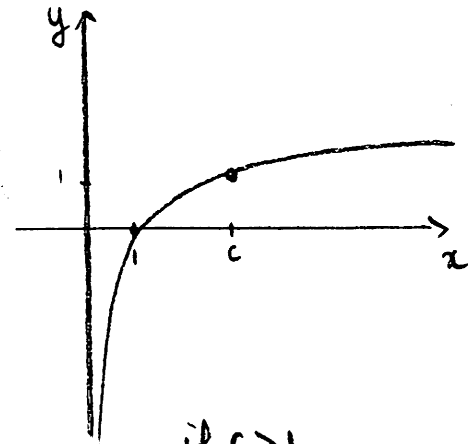


if $c>1$

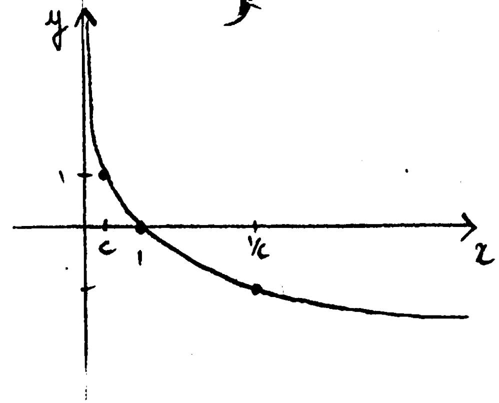


if $0<c<1$

LOGARITHMIC FUNCTIONS



if $c>1$



if $0<c<1$

GRAPHS and CHARACTERISTICS

Domain: $\mathbb{R}$ (no restrictions!) → not affected by transf.

Range: $(0;+\infty)$ → affected by vertical translat°

Asymptote: $y=0$ (horiz) → affected by vertical translat°

y-int: 1 → affected by all transf. except. horiz stretch

x-int: none → unless vert. translat°

increasing or decreasing?

increasing ↑ when $c>1$

decreasing ↑ when $0<c<1$

$D = (0;+\infty)$ (⚠ restrictions!) → affected by horiz transl

$R = \mathbb{R}$ → not affected

Asymptote: $x=0$ (vert.) → affected by horiz. transl

y-int: none → affected by horiz. transl.

x-int: 1 → affected by all except vert. stretches.

increasing when $c>1$
decreasing when $0<c<1$

$c \neq 1$

Exponential form $\leftrightarrow$ log. form

$$y = \log_c x \iff c^y = x$$

⚠ To change form you need to isolate your exp or log function!

ex: $3x+5 = 2\log(x-1) - 4$

$3x+9 = 2\log(x-1)$

$\frac{3x+9}{2} = \log(x-1) \Rightarrow 10^{\frac{3x+9}{2}} = x-1$

PROPERTIES

Numerical values

* You can find an exact value for $\log_b a$ as long as b and a can be expressed on the same base!

Ex: • $\log_3 27 = 3$ because $3^{\boxed{3}} = 27$

• $\log_4 8 = \frac{3}{2}$ because $4^x = 8$
$2^{2x} = 2^3$
$2x = 3$

* You can express any number as a log:

Ex: • $1 = \log 10$

• $2 = \log_3 9$

Log laws and when to use them

* $\log_2(16x) = \log_2 16 + \log_2 x$

$\Rightarrow$ useful to isolate a variable

$\Rightarrow$ useful to break down into logs that we know

ex: $\log_3 27\sqrt{3} = \log_3 27 + \log_3 \sqrt{3}$
$= 3 + \frac{1}{2}$
$= \frac{7}{2}$

* $\log(x+3) + \log(x-4) = \log(x+3)(x-4)$

$\Rightarrow$ useful to regroup terms when solving equations (because logs need to be isolated!)

$\Rightarrow$ useful to simplify expressions sometimes.

ex: $\log_{12} 24 - \log_{12} 6 + \log_{12} 36$
$= \log_{12} \frac{24 \times 36}{6}$
$= \log_{12} 144$
$= 2$

* $\log(3^{x-1}) = (x-1)\log 3$

$\Rightarrow$ useful to reach an exponent when solving equations.

* $2\log 3 = \log 3^2$

$\Rightarrow$ useful to isolate a log when solving an equation or moving to exp. form.

SOLVING EQUATIONS

PONENTIAL EQUATIONS

- No Restrictions on the domain (except for "real life situations")

1. Try writing with the same base!

Ex

$$27^{3x} = 9^{2x-4}$$

$$\hookrightarrow 3^{3\times 3x} = 3^{2(2x-4)}$$

$$9x = 4x - 8 \quad ...$$

2. Compose by log on both sides

Ex 1

$$3^{2x-5} = 5^{x+1}$$

$$\hookrightarrow (2x-5)\log 3 = (x+1)\log 5$$

$$2x\log 3 - 5\log 3 = x\log 5 + \log 5$$

$$2x\log 3 - x\log 5 = 5\log 3 + \log 5$$

$$x(2\log 3 - \log 5) = 5\log 3 + \log 5$$

$$x = \frac{5\log 3 + \log 5}{2\log 3 - \log 5}$$

exact value

Ex 2

$$3^{2x-5} = 4$$

$$(2x-5)\log 3 = \log 4$$

$$2x\log 3 - 5\log 3 = \log 4$$

$$2x\log 3 = \log 4 + 5\log 3$$

$$x = \frac{\log 4 + 5\log 3}{2\log 3}$$

3. Graphically

$$3^{2x-5} = 4$$

graph $\left.\begin{array}{l} y = 3^{2x-5} \\ \& \; y = 4 \end{array}\right\}$ and look at the x-coord of the intersection.

LOGARITHMIC EQUATIONS

1. ⚠ Always start by determining RESTRICTIONS!!
2. Isolate your log function(s).
3. check restrictions!

$\hookrightarrow$ $\boxed{\log_c \Delta = \square \text{ or } \log_c \Delta = \log_c \square}$

Examples

Ex 1

$$2\log_2(x+3) = 4$$

$\hookrightarrow$ Restr: $x+3>0 \qquad x>-3$

Resol: $\log_2(x+3) = 2$

$$2^2 = x+3$$

$$x = 1 \qquad \text{Restr} \checkmark$$

Ex 2

$$\log(8x+4) = 1 + \log(x+1)$$

$\hookrightarrow$ Restr: $8x+4>0$, $x>-\frac{1}{2}$ and $x+1>0$, $x>-1$ $\}$ $\boxed{x>-\frac{1}{2}}$

Resol: $\log(8x+4) = \log 10 + \log(x+1)$

$$\log(8x+4) = \log 10(x+1)$$

$$8x+4 = 10(x+1)$$

$$-6 = 2x$$

$$\cancel{x = -3}$$

Restr $\times$

banking 9^{o} (#11 p365) for example

\$1000 invested @ 8% per year compounded quarterly

$\Rightarrow A = 1000 \times 1.02^{4t}$

After 5 years:

$A \simeq \$1485.95$

The pH of a solution is defined as pH = −log [H⁺], where [H⁺] is the hydrogen ion concentration, in moles per litre (mol/L). Acetic acid has a pH of 2.9. Formic acid is 4 times as concentrated as acetic acid. What is the pH of formic acid?

$pH_A = -\log[H^+]_A$

$pH_F = -\log[H^+]_F$

$pH_A = 2.9$

$[H^+]_F = 4[H^+]_A$

$$pH_F = -\log[H^+]_F$$
$$= -\log(4[H^+]_A)$$
$$= -\log 4 - \log[H^+]_A$$
$$= -\log 4 + 2.9$$

$pH_F \simeq 2.3$

"FAMOUS" Word Problems

The Richter magnitude, M, of an earthquake is defined as $M = \log\left(\frac{A}{A_0}\right)$, where A is the amplitude of the ground motion and A_0 is the amplitude, corrected for the distance to the actual earthquake, that would be expected for a standard earthquake. An earthquake near Tofino, British Columbia, measures 5.6 on the Richter scale. An aftershock is $\frac{1}{4}$ the intensity of the original earthquake. Determine the magnitude of the aftershock on the Richter scale, to the nearest tenth.

↳ $M_T = \log\frac{A_T}{A_0}$ $M_A = \log\frac{A_A}{A_0}$

$M_T = 5.6$

$A_A = \frac{1}{4}A_T$

$$M_A = \log\frac{A_A}{A_0}$$
$$= \log\frac{\frac{1}{4}A_T}{A_0}$$
$$= \log\frac{1}{4} + \log\frac{A_T}{A_0}$$
$$= \log\frac{1}{4} + 5.6$$

$\boxed{M_A \simeq 5.0}$

from quiz

(see answer key)