SOLUTIONS

Precalc 12 - PRACTICE EXAM

CALCULATOR ALLOWED

1. Solve the following equations.

Give exact values when possible. If not possible, then round to the nearest hundredth.

a) $5^{3x-2} = 25^{x+1}$

$5^{3x-2} = 5^{2(x+1)}$

$5^{3x-2} = 5^{2x+2}$

$3x - 2 = 2x + 2$

$\boxed{x = 4}$

b) $8^{4x+1} = 16^{x-3}$

$2^{3(4x+1)} = 2^{4(x-3)}$

$12x + 3 = 4x - 12$

$8x = -15$

$\boxed{x = -\frac{15}{8}}$

c) $3^{x-3} = 4^{2x+1}$

$\ln(3^{x-3}) = \ln(4^{2x+1})$

$(x-3)\ln 3 = (2x+1)\ln 4$

$x\ln 3 - 3\ln 3 = 2x\ln 4 + \ln 4$

$x\ln 3 - 2x\ln 4 = 3\ln 3 + \ln 4$

$x(\ln 3 - 2\ln 4) = 3\ln 3 + \ln 4$

$\boxed{x = \frac{3\ln 3 + \ln 4}{\ln 3 - 2\ln 4}}$

d) $5^{3x-1} = 7^{x-6}$

$(3x-1)\ln 5 = (x-6)\ln 7$

$3x\ln 5 - \ln 5 = x\ln 7 - 6\ln 7$

$3x\ln 5 - x\ln 7 = \ln 5 - 6\ln 7$

$x(3\ln 5 - \ln 7) = \ln 5 - 6\ln 7$

$\boxed{x = \frac{\ln 5 - 6\ln 7}{3\ln 5 - \ln 7}}$

e) $log_4(x-3) = 2 - \log_4(x+3)$

- restrictions:

$x - 3 > 0$ and $x + 3 > 0$

$x > 3$ $\quad$ $x > -3$

$\boxed{x > 3}$

- resolution:

$\log_4(x-3) + \log_4(x+3) = 2$

$\log_4(x-3)(x+3) = 2$

$4^2 = (x-3)(x+3)$

$16 = x^2 - 9$

$x^2 = 25$

$x = \pm 5$ ⚠ restr $\boxed{x = 5}$

f) $\log(3x + 6) = 1 + logx$

- restrictions:

$3x + 6 > 0$ and $x > 0$

$x > -2$

$\boxed{x > 0}$

- resolution

$\log(3x+6) = \log 10 + \log x$

$\log(3x+6) = \log 10x$

$3x + 6 = 10x$

$7x = 6$

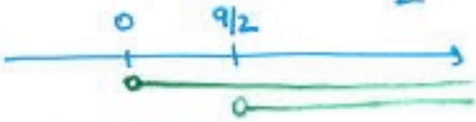
$\boxed{x = \frac{6}{7}}$ Restr ✓

g) $\log_5 x + \log_5(2x - 9) - 1 = 0$

- restrictions:

$x > 0$ and $2x - 9 > 0$

$x > \frac{9}{2}$



$\boxed{x > \frac{9}{2}}$

- resolution

$\log_5 x + \log_5(2x - 9) = 1$

$\log_5 x(2x-9) = 1$

$5^1 = x(2x-9)$

$5 = 2x^2 - 9x$

$2x^2 - 9x - 5 = 0$

$(x-5)(2x+1) = 0$

$\boxed{x = 5}$ $\quad$ $\cancel{x = -\frac{1}{2}}$ Restr ✗

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h) $\cos\theta = -\frac{1}{2}$ for $-\pi \le \theta \le 2\pi$

solutions : $\left\{-\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

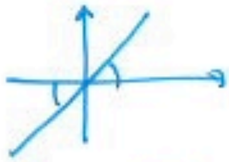
$\theta_R = \pi/3$

i) $\sin\theta = -\frac{1}{\sqrt{2}}$ for $-2\pi \le \theta \le \pi$

solutions : $\left\{-\frac{3\pi}{4}, -\frac{\pi}{4}\right\}$

$\theta_R = \frac{\pi}{4}$

j) $\tan\theta = 1$ (general solution in radians)



solutions : $\left\{\frac{\pi}{4} + n\pi, n \in \mathbb{Z}\right\}$

$\theta_R = \frac{\pi}{4}$

k) $3\tan^2\theta = 1$ for $-\pi \le \theta \le 2\pi$

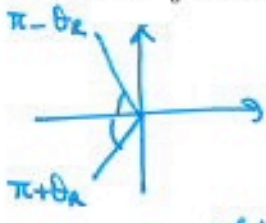
$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$

solutions : $\left\{-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

$\theta_R = \pi/6$

l) $\cos\theta = -\frac{1}{3}$ for $0 \le \theta \le 2\pi$



solutions : $\{1.91 ; 4.37\}$

$\theta_R = \cos^{-1}\left(\frac{1}{3}\right) \simeq 1.23$

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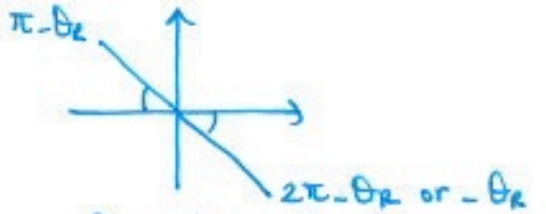
m) $\tan\theta = -0.3$ for $-\pi \le \theta \le 2\pi$



$\theta_R = \tan^{-1}(0.3) \simeq 0.29$

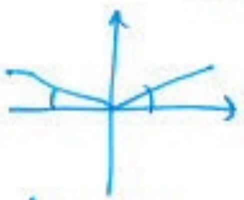
solutions : $\{-0.29, 2.85, 5.99\}$

n) $2\sin\left(3x + \frac{\pi}{4}\right) = 1$ for $0 \le x \le 2$

let $\theta = 3x + \frac{\pi}{4}$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$



$\theta_R = \pi/6$

$3x + \frac{\pi}{4} = \frac{\pi}{6}$

$3x = -\frac{\pi}{12}$

$\cancel{x = -\frac{\pi}{36} + \frac{2\pi}{3}n}$

no solution in the domain

$3x + \frac{\pi}{4} = \frac{5\pi}{6}$

$3x = \frac{7\pi}{12}$

$x = \frac{7\pi}{36} + \frac{2\pi}{3}n, \ n \in \mathbb{Z}$

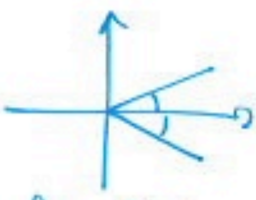
solution : $\left\{\frac{7\pi}{36}\right\}$

p) $2\cos\left(\frac{\pi}{3}(x+1)\right) = \sqrt{3}$ for $0 \le x \le 20$

let $\theta = \frac{\pi}{3}(x+1)$

$2\cos\theta = \sqrt{3}$

$\cos\theta = \frac{\sqrt{3}}{2}$



$\theta_R = \frac{\pi}{6}$

$\frac{\pi}{3}(x+1) = \frac{\pi}{6}$

$x + 1 = \frac{1}{2}$

$x = -\frac{1}{2} + 6n$

$\frac{\pi}{3}(x+1) = -\frac{\pi}{6}$

$x + 1 = -\frac{1}{2}$

$x = -\frac{3}{2} + 6n, \ n \in \mathbb{Z}$

solutions : $\{4.5, 5.5, 10.5, 11.5, 16.5, 17.5\}$

$P = \frac{2\pi}{\frac{\pi}{3}} = 2\pi \times \frac{3}{\pi} = 6$

q) $2x^3 - 2x^2 + 26x + 31 = (3x + 4)^2$

$2x^3 - 2x^2 + 26x + 31 = 9x^2 + 24x + 16$

$2x^3 - 11x^2 + 2x + 15 = 0$

$(x-5)(x+1)(2x-3) = 0$

$x - 5 = 0$ $\quad$ $x + 1 = 0$ $\quad$ $2x - 3 = 0$

$x = 5$ $\quad$ $x = -1$ $\quad$ $x = \frac{3}{2}$

solutions : $\{-1, \frac{3}{2}, 5\}$

r) $2(x^4 + 4x^3 - 5x) = x^3 - x^2$

$2x^4 + 8x^3 - 10x - x^3 + x^2 = 0$

$2x^4 + 7x^3 + x^2 - 10x = 0$

$x(2x^3 + 7x^2 + x - 10) = 0$

$x(x-1)(x+2)(2x+5) = 0$

$x = 0$ or $x = 1$ or $x = -2$ or $x = -5/2$

Solutions: $\{-\frac{5}{2}, -2, 0, 1\}$

Precalc 12 - PRACTICE EXAM

s) $\frac{4}{x^2-1} - \frac{x-1}{x+1} = \frac{x+7}{x-1}$

- restrictions

$x \neq \pm 1$

- resolution:

$4 - (x-1)(x-1) = (x+7)(x+1)$

$4 - (x^2 - 2x + 1) = x^2 + 8x + 7$

$4 - x^2 + 2x - 1 = x^2 + 8x + 7$

$2x^2 + 6x + 4 = 0$

$2(x+1)(x+2) = 0$

$\cancel{x = -1}$ Restr$_x$ or $\boxed{x = -2}$

t) $2 - \frac{1}{x^2+x} = \frac{3}{x+1}$

- restrictions

$x(x+1) \neq 0 \quad x+1 \neq 0$

$x \neq 0$ and $x \neq -1$

- resolution:

$2x(x+1) - 1 = 3x$

$2x^2 + 2x - 1 - 3x = 0$

$2x^2 - x - 1 = 0$

$\boxed{x = 1 \text{ or } x = -\frac{1}{2}}$

u) $\frac{x}{x-5} + \frac{3}{x+2} = \frac{7x}{x^2-3x-10}$

- restrictions:

$x \neq 5$ & $x \neq -2$

- resolution:

$x(x+2) + 3(x-5) = 7x$

$x^2 + 2x + 3x - 15 = 7x$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$\cancel{x = 5}$ Restr$_x$ $\boxed{x = -3}$

Precalc 12 - PRACTICE EXAM

NON-CALCULATOR SECTION

2. Let $f(x) = 3\sqrt{-2x+1} - 5$

a) Describe what transformations can be applied to the graph of $y = \sqrt{x}$ to get the graph of $y = f(x)$.

$f(x) = 3\sqrt{-2(x - \frac{1}{2})} - 5$

- vertical stretch factor 3
- horiz. reflection
- horiz stretch factor 1/2
- horiz. translation $\frac{1}{2} \rightarrow$
- vertical transl 5 $\downarrow$

b) Determine the domain and the range of f.

$-2(x - \frac{1}{2}) \geq 0$

$x - \frac{1}{2} \leq 0$

$\boxed{x \leq \frac{1}{2}}$

$D = (-\infty, \frac{1}{2}]$

$R = [-5, +\infty)$

c) Determine the inverse of f and state its domain and its range.

$x = 3\sqrt{-2y+1} - 5$

$x + 5 = 3\sqrt{-2y+1}$

$\frac{x+5}{3} = \sqrt{-2y+1}$

$-2y + 1 = \left(\frac{x+5}{3}\right)^2$

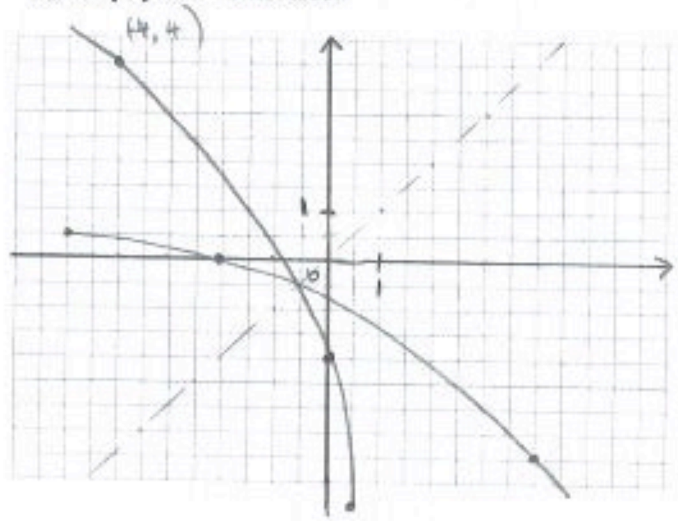
$-2y = \left(\frac{x+5}{3}\right)^2 - 1$

$\boxed{y = -\frac{1}{2}\left(\frac{x+5}{3}\right)^2 + \frac{1}{2}}$

$D = [-5, +\infty)$

$R = (-\infty, \frac{1}{2}]$

d) Graph f and its inverse.



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3. Let $g(x) = x^2 - 2x - 3$

a) Describe what transformations can be applied to the graph of $y = x^2$ to get the graph of $y = g(x)$.

$g(x) = (x-1)^2 - 4$

- translation 1 →
- . . . . 4 ↓

b) Determine the domain and the range of g.

$D = \mathbb{R}$ $R = [-4, +\infty)$

c) Determine the inverse of g and state its domain and its range.

$x = (y-1)^2 - 4$

$x + 4 = (y-1)^2$

$\pm\sqrt{x+4} = y - 1$

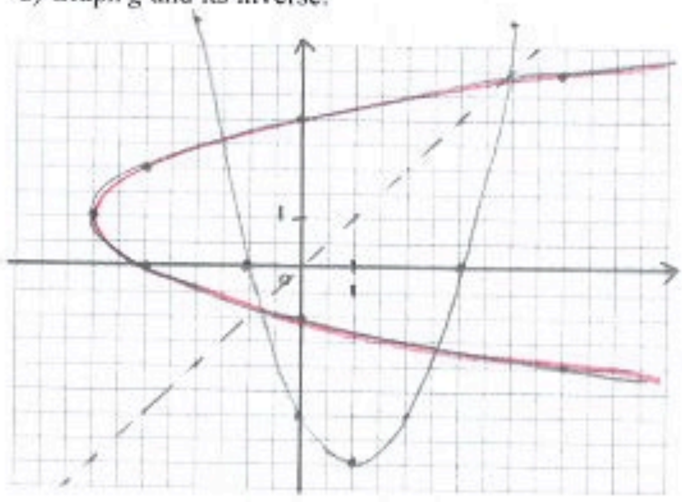
$y = \pm\sqrt{x+4} + 1$

$D = [-4, +\infty)$

$R = \mathbb{R}$ (if the original is not restricted)

both functions combined

d) Graph g and its inverse.



e) Restrict the domain of g so that its inverse is a function.

$D_R = [1, +\infty)$

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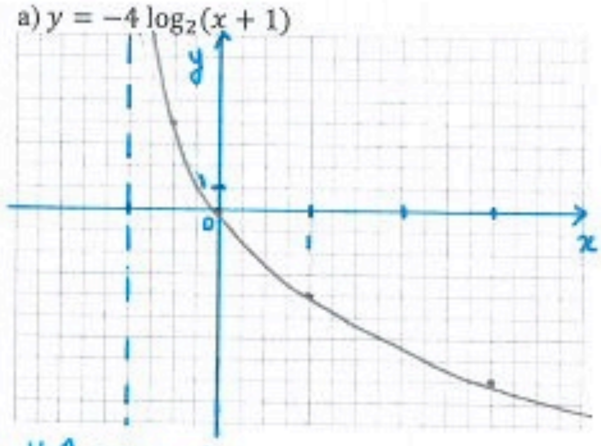
4. Graph the following functions and their asymptotes :

a) $y = -4\log_2(x+1)$



V.A : $x = -1$

$y = 2^x$

x	y
-1	1/2
0	1
1	2
2	4

$y = \log_2 x$

x	y
1/2	-1
1	0
2	1
4	2

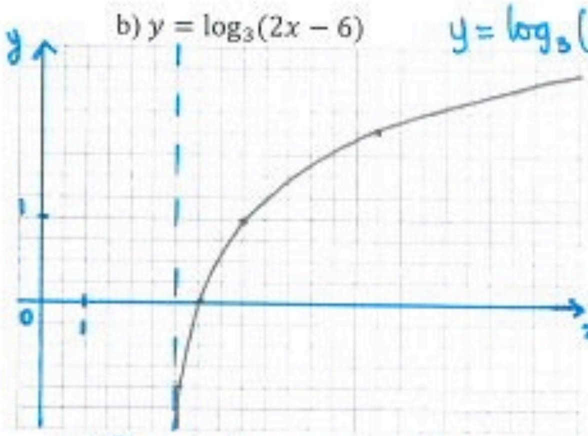
$y = -4\log_2 x$

x	y
1/2	4
1	0
2	-4
4	-8

$y = -4\log_2(x+1)$

x	y
-1/2	4
0	0
1	-4
3	-8

b) $y = \log_3(2x-6)$ $y = \log_3(2(x-3))$



$y = 3^x$

x	y
-1	1/3
0	1
1	3
2	9

$y = \log_3 x$

x	y
1/3	-1
1	0
3	1
9	2

$y = \log_3 2x$

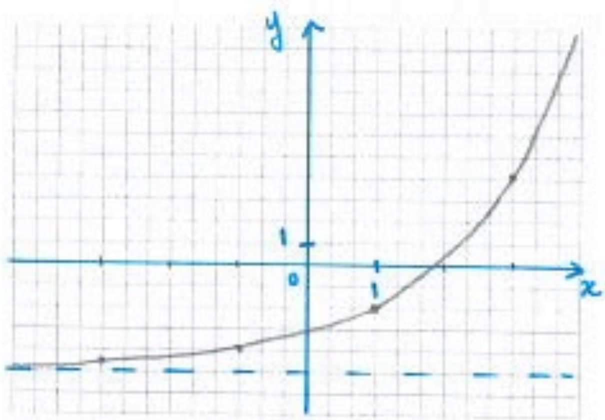
x	y
1/6	-1
1/2	0
3/2	1
9/2	2

$y = \log_3 2(x-$

x	y
3+1/6	-1
3.5	0
4.5	1
7.5	2

V.A : $x = 3$

a) $y = 3^{\frac{1}{2}(x+1)} - 5$



$y = 3^x \xrightarrow{\text{str}}$ $\xrightarrow{\text{tran.}}$

x	y
-1	1/3
0	1
1	3
2	9

$x \times 2$

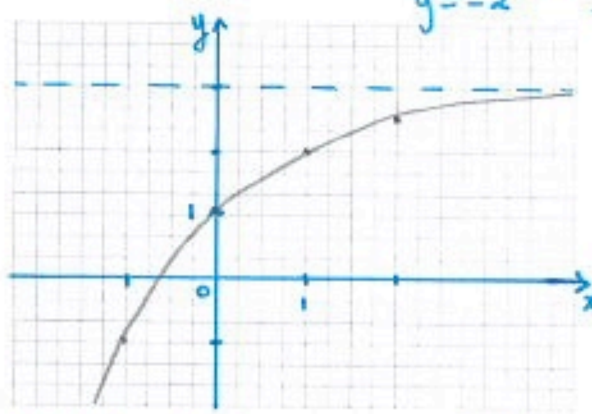
x	y
-2	1/3
0	1
2	3
4	9

$x - 1$
$y - 5$

x	y
-3	-4.6
-1	-4
1	-2
3	4

H.A $y = -5$

b) $y = -2^{-x+1} + 3$ $y = -2^{-(x-1)} + 3$



$y = 2^x \xrightarrow[\text{refl}]{\text{str}}$ $\xrightarrow{\text{trans}}$

x	y
-1	1/2
0	1
1	2
2	4

$y \times (-1)$
$x \times (-1)$

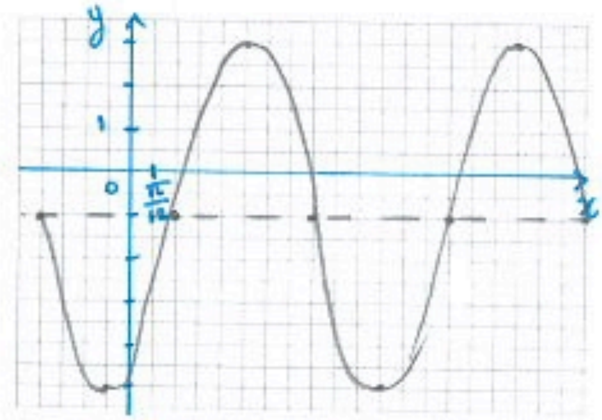
x	y
1	-1/2
0	-1
-1	-2
-2	-4

$x + 1$
$y + 3$

x	y
2	2.5
1	2
0	1
-1	-1

H.A : $y = 3$

c) $y = 4\sin\left(2\left(x - \frac{\pi}{6}\right)\right) - 1$



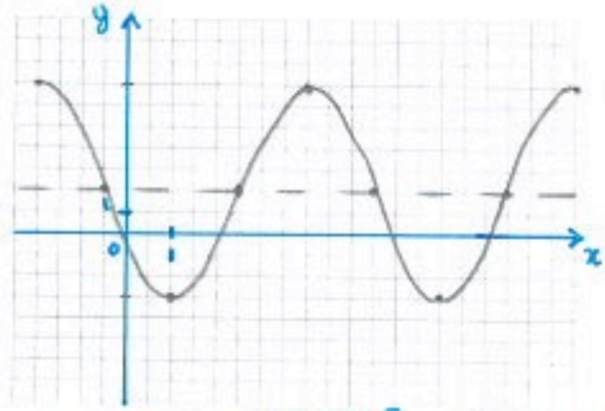
$a = 4$ → max: 3, min: −5

$b = 2 \Rightarrow$ period: π (pt every $\frac{\pi}{4}$)

$h = \frac{\pi}{6}$ (scale $\frac{\pi}{12}$)

$k = -1$ (centre line)

c) $y = -5\cos\left(\frac{\pi}{3}(x-1)\right) + 2$



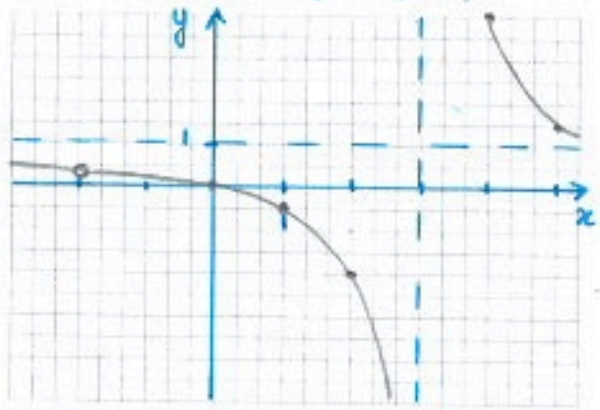
$a = -5$ → max: 7, min: −3

$b = \pi/3 \rightarrow$ period $= 6$ (pt every 1.5)

$h = 1$

$k = 2$

c) $y = \frac{2x^2+4x}{2x^2-2x-12} = \frac{2x(x+2)}{2(x-3)(x+2)}$



V.A: $x = 3$

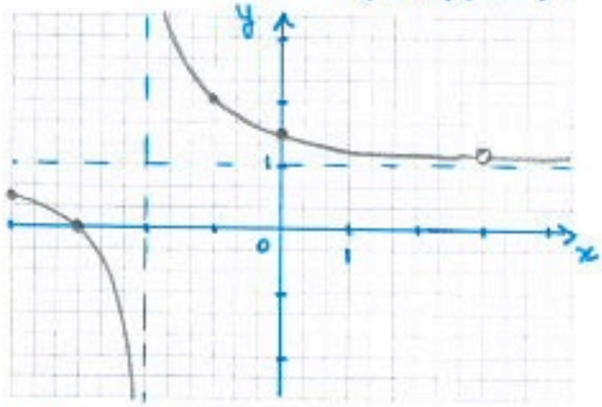
hole: $(-2, 0.4)$

H.A: $y = 1$

y-int: 0

x-int: 0

c) $y = \frac{x^2-9}{x^2-x-6} = \frac{(x+3)(x-3)}{(x-3)(x+2)}$



V.A: $x = -2$

hole: $(3, 1.2)$

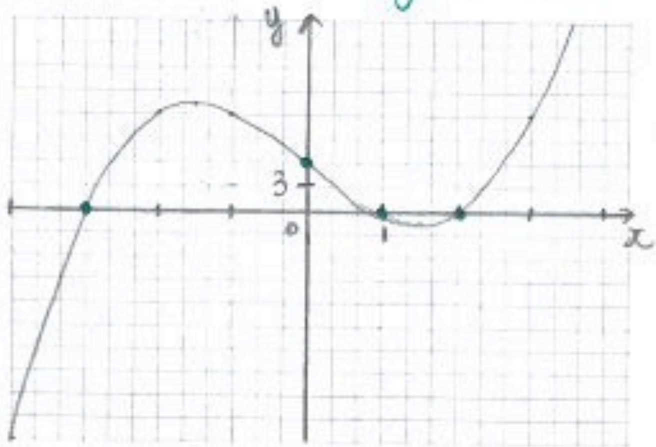
H.A: $y = 1$

y-int: $\frac{3}{2}$

x-int: -3

d) $y = x^3 - 7x + 6$

$y = (x-2)(x-1)(x+3)$

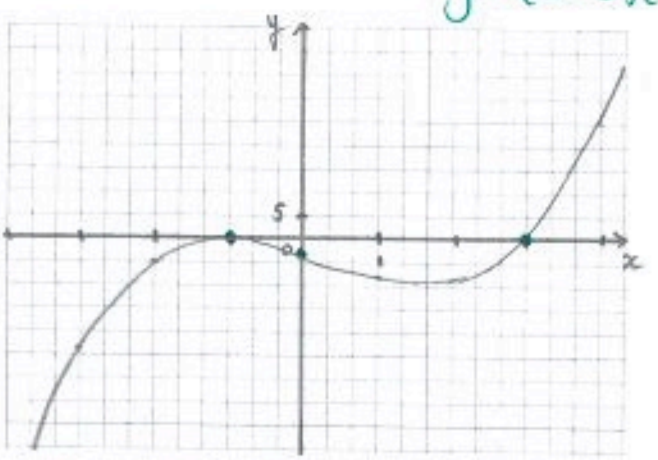


Zeros: -3, 1, 2

y-int: 6

d) $y = x^3 - x^2 - 5x - 3$

$y = (x-3)(x+1)^2$



Zeros: -1 (mult 2)
3

y-int: -3

5. Expand and simplify the following expressions :

a) $\log 5x(x+1)^2 + \log\frac{500}{x+1} - \log x(x-3)$

$= \log 5 + \cancel{\log x} + 2\log(x+1) + \log 500 - \log(x+1) - \cancel{\log x} - \log(x-3)$

$= \log(x+1) - \log(x-3) + \log 5 + \log 5 + \log 100$

$= \log(x+1) - \log(x-3) + 2\log 5 + 2$

a) $\ln\left(\frac{3xe^{2x}}{x^2-3x}\right) + \ln(x-3)$

$= \ln 3 + \ln x + 2x - \ln(x(x-3)) + \ln(x-3)$

$= \ln 3 + \cancel{\ln x} + 2x - \cancel{\ln x} - \cancel{\ln(x-3)} + \cancel{\ln(x-3)}$

$= \ln 3 + 2x$

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6. Prove the following identities :

a) $\frac{1}{\sin x+1}-\frac{1}{\sin x-1}=\frac{2}{\cos^2 x}$

$$= \frac{\sin x - 1 - (\sin x+1)}{(\sin x+1)(\sin x-1)}$$

$$= \frac{-2}{\sin^2 x - 1}$$

$$= \frac{2}{1-\sin^2 x}$$

$$= \frac{2}{\cos^2 x} \checkmark$$

b) $\frac{\tan x}{\sec x-1}=\frac{\sec x+1}{\tan x}$

$$= \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}-1}$$

$$= \frac{\frac{\sin x}{\cos x}}{\frac{1-\cos x}{\cos x}}$$

$$= \frac{\sin x}{\cancel{\cos x}} \cdot \frac{\cos x}{1-\cos x}$$

$$\frac{\sin x}{1-\cos x} \checkmark$$

$$= \frac{\frac{1}{\cos x}+1}{\frac{\sin x}{\cos x}}$$

$$= \frac{\frac{1+\cos x}{\cos x}}{\frac{\sin x}{\cos x}}$$

$$= \frac{1+\cos x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{\sin x}$$

$$\frac{1+\cos x}{\sin x} \cdot \frac{1-\cos x}{1-\cos x}$$

$$= \frac{1-\cos^2 x}{\sin x(1-\cos x)}$$

$$= \frac{\sin^{\cancel{2}} x}{\cancel{\sin x}(1-\cos x)}$$

$$= \frac{\sin x}{1-\cos x} \checkmark$$

c) $\frac{\sec x+\csc x}{\tan x+\cot x}=\sin x+\cos x$

$$= \frac{\frac{1}{\cos x}+\frac{1}{\sin x}}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}$$

$$= \frac{\frac{\sin x+\cos x}{\cos x \sin x}}{\frac{\sin^2 x+\cos^2 x}{\cos x \sin x}}$$

$$= \frac{\frac{\sin x+\cos x}{\cos x \sin x}}{\frac{1}{\cos x \cdot \sin x}}$$

$$\frac{\sin x+\cos x}{\cancel{\cos x \cdot \sin x}} \cdot \frac{\cancel{\cos x \cdot \sin x}}{1}$$

$$\sin x + \cos x \checkmark$$