

QUIZ 6.1-6.2

1. Determine the non-permissible values of x, in radians, for each expression.

a) $\frac{\sin x}{\cos x}$ • $\cos x \neq 0$



$x \neq \frac{\pi}{2} + n\pi, n \in \mathbb{I}$

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b) $\frac{\cos x}{\tan x}$ • $\cos x \neq 0$ • $\sin x \neq 0$



$x \neq 0 + n\frac{\pi}{2}, n \in \mathbb{I}$

2

c) $\frac{\cot x}{1 + \sin x}$ • $\sin x \neq 0$ • $\sin x \neq -1$



$x \neq 0 + n\pi, n \in \mathbb{I}$
 $x \neq -\frac{\pi}{2} + 2n\pi, n \in \mathbb{I}$

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2. Simplify each expressions to one of the 3 primary trigonometric functions, sinx, cosx, or tanx (without finding the non-permissible values).

a) $\cot x \cdot \sin x = \frac{\cos x}{\sin x} \cdot \sin x = \cos x$

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b) $\frac{\sec^2 x \cdot \cos x}{\csc x} = \frac{\frac{1}{\cos^2 x} \cdot \cos x}{\frac{1}{\sin x}} = \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} = \frac{1}{\cos x} \times \frac{\sin x}{1} = \tan x$

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c) $\frac{\cot x \cdot \tan x}{\csc x} = \frac{\frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x}}{\frac{1}{\sin x}} = \sin x$

2

$$d) \csc x - \frac{\cot x}{\sec x} = \frac{1}{\sin x} - \frac{\frac{\cos x}{\sin x}}{\frac{1}{\cos x}} = \frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}$$

$$= \frac{1 - \cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} = \boxed{\sin x}$$

2

3. Verify that the equation $\frac{\csc x}{\tan x + \cot x} = \cos x$ is true for $x = 60^\circ$ and for $x = \frac{\pi}{6}$.

$$x = 60^\circ \quad \text{L.S.} \quad \frac{\csc 60}{\tan 60 + \cot 60} = \frac{\frac{2}{\sqrt{3}}}{\sqrt{3} + \frac{1}{\sqrt{3}}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{\sqrt{3}}} = \frac{1}{2} \quad \text{R.S.:} \quad \cos 60 = \frac{1}{2} \quad \checkmark$$

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$$x = \frac{\pi}{6} \quad \text{L.S.:} \quad \frac{\csc \frac{\pi}{6}}{\tan \frac{\pi}{6} + \cot \frac{\pi}{6}} = \frac{2}{\frac{1}{\sqrt{3}} + \sqrt{3}} = \frac{2}{\frac{4}{\sqrt{3}}} = \frac{\sqrt{3}}{2} \quad \text{R.S.:} \quad \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \quad \checkmark$$

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4. Determine the restrictions, and prove the following identity:

$$\tan x + \frac{1}{\tan x} = \frac{1}{\cos x \cdot \sin x}$$

• Restrictions: $\sin x \neq 0$ $\cos x \neq 0$

$$\boxed{x \neq 0 + n\frac{\pi}{2}, n \in \mathbb{I}}$$

$$\text{• L.S.:} \quad \tan x + \frac{1}{\tan x} = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x}$$

$$= \frac{1}{\cos x \cdot \sin x} \quad \text{R.S.} \quad \checkmark$$

2

5. Write each expression as a single trigonometric function :

a) $\sin 72^\circ \cdot \cos 46^\circ - \cos 72^\circ \cdot \sin 46^\circ = \sin(72^\circ - 46^\circ)$
 $= \boxed{\sin 26^\circ}$

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b) $\frac{\tan 28^\circ + \tan 33^\circ}{1 - \tan 28^\circ \cdot \tan 33^\circ} = \tan(28^\circ + 33^\circ)$
 $= \boxed{\tan 61^\circ}$

2

c) $1 - 2\sin^2 \frac{\pi}{8} = \cos\left(2 \times \frac{\pi}{8}\right)$
 $= \boxed{\cos \frac{\pi}{4}} = \frac{1}{\sqrt{2}}$

2

d) $\frac{\cos 2x - 1}{\sin 2x} = \frac{1 - 2\sin^2 x - 1}{2\sin x \cdot \cos x}$
 $= -\frac{\sin x}{\cos x}$
 $= \boxed{-\tan x}$

2

e) $8\sin^2 2\theta - 4 = 4(2\sin^2 2\theta - 1)$
 $= \boxed{-4 \cdot \cos 4\theta}$

2

6. Give the exact value for each expression :

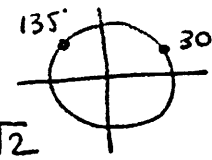
a) $\frac{2 \tan \frac{\pi}{3}}{1 - \tan^2 \frac{\pi}{3}} = \frac{2\sqrt{3}}{1 - \sqrt{3}^2} = \frac{2\sqrt{3}}{-2} = \boxed{-\sqrt{3}}$ or $\tan \frac{2\pi}{3} \dots$

2

b) $\cos \frac{\pi}{3} \cdot \cos \frac{\pi}{12} + \sin \frac{\pi}{3} \cdot \sin \frac{\pi}{12} = \cos \left(\frac{\pi}{3} - \frac{\pi}{12} \right)$
 $= \cos \frac{3\pi}{12}$
 $= \cos \frac{\pi}{4} = \boxed{\frac{1}{\sqrt{2}}}$

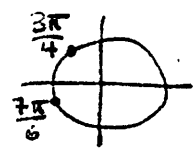
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c) $\sin 105^\circ = \sin (135^\circ - 30^\circ)$
 $= \sin 135^\circ \cdot \cos 30^\circ - \cos 135^\circ \cdot \sin 30^\circ$
 $= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{\sqrt{6}+\sqrt{2}}{4}$



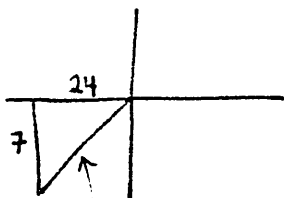
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d) $\tan \frac{23\pi}{12} = \tan \left(\frac{7\pi}{6} + \frac{3\pi}{4} \right)$
 $= \frac{\tan \frac{7\pi}{6} + \tan \frac{3\pi}{4}}{1 - \tan \frac{7\pi}{6} \cdot \tan \frac{3\pi}{4}}$
 $= \frac{\frac{1}{\sqrt{3}} + (-1)}{1 - \frac{1}{\sqrt{3}} \times (-1)} = \frac{\frac{1-\sqrt{3}}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{1-\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}+1} = \boxed{\frac{1-\sqrt{3}}{\sqrt{3}+1}} = \sqrt{3} - 2$



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7. Angle θ is in quadrant III and $\tan \theta = \frac{7}{24}$. Determine an exact value for $\sin 2\theta$.

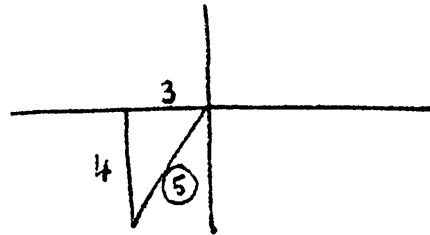
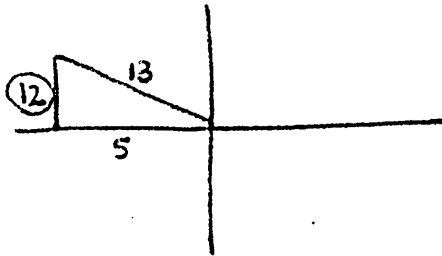


$\sin 2\theta = 2 \sin \theta \cdot \cos \theta$
 $= 2 \times \left(-\frac{7}{25} \right) \times \left(-\frac{24}{25} \right)$
 $= \boxed{\frac{336}{625}}$

2

25
 (Pythagorean Th.)

8. Angle x is in quadrant II, angle y is in quadrant III, $\cos x = -\frac{5}{13}$, and $\tan y = \frac{4}{3}$.
Determine the value of $\cos(x - y)$.



$$\cos(x - y) = \cos x \cdot \cos y + \sin x \cdot \sin y$$

$$= -\frac{5}{13} \cdot \left(-\frac{3}{5}\right) + \frac{12}{13} \cdot \left(-\frac{4}{5}\right)$$

$$= \boxed{-\frac{33}{65}}$$

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