Key Ideas

- A geometric sequence is a sequence in which each term, after the first term, is found by multiplying the previous term by a non-zero constant, r, called the common ratio.
- The common ratio of successive terms of a geometric sequence can be found by dividing any two consecutive terms, $r = \frac{t_n}{t_{n-1}}$.
- The general term of a geometric sequence is
  $t_n = t_1 r^{n-1}$
  where t_1 is the first term
  n is the number of terms
  r is the common ratio
  t_n is the general term or nth term

Check Your Understanding

Practise

1. Determine if the sequence is geometric. If it is, state the common ratio and the general term in the form $t_n = t_1 r^{n-1}$.

a) 1, 2, 4, 8, …

b) 2, 4, 6, 8, …

c) 3, −9, 27, −81, …

d) 1, 1, 2, 4, 8, …

e) 10, 15, 22.5, 33.75, …

f) −1, −5, −25, −125, …

2. Copy and complete the following table for the given geometric sequences.

	Geometric Sequence	Common Ratio	6th Term	10th Term
a)	6, 18, 54, …			
b)	1.28, 0.64, 0.32, …			
c)	$\frac{1}{5}, \frac{3}{5}, \frac{9}{5}, \ldots$			

3. Determine the first four terms of each geometric sequence.

a) $t_1 = 2, r = 3$

b) $t_1 = -3, r = -4$

c) $t_1 = 4, r = -3$

d) $t_1 = 2, r = 0.5$

4. Determine the missing terms, t_2, t_3, and t_4, in the geometric sequence in which $t_1 = 8.1$ and $t_5 = 240.1$.

5. Determine a formula for the nth term of each geometric sequence.

a) $r = 2, t_1 = 3$

b) 192, −48, 12, −3, …

c) $t_3 = 5, t_6 = 135$

d) $t_1 = 4, t_{13} = 16\ 384$

Apply

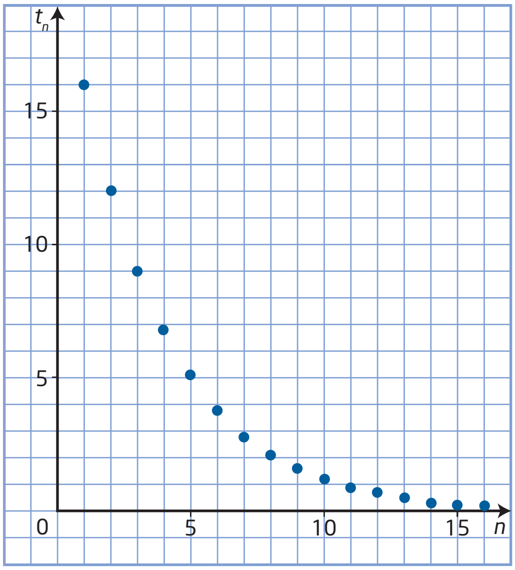
6. Given the following geometric sequences, determine the number of terms, n.

	Table A			
	First Term, t_1	Common Ratio, r	nth Term, t_n	Number of Terms, n
a)	5	3	135	
b)	−2	−3	−1458	
c)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{48}$	
d)	4	4	4096	
e)	$-\frac{1}{6}$	2	$-\frac{128}{3}$	
f)	$\frac{p^2}{2}$	$\frac{p}{2}$	$\frac{p^9}{256}$	

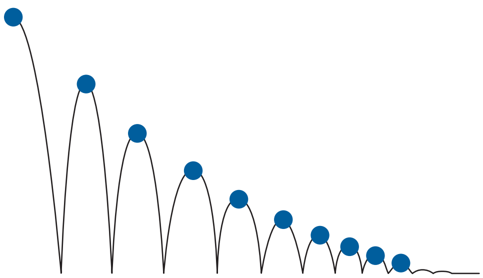
7. The following sequence is geometric. What is the value of y?

3, 12, 48, $5y + 7$, …

8. The following graph illustrates a geometric sequence. List the first three terms for the sequence and state the general term that describes the sequence.



9. A ball is dropped from a height of 3.0 m. After each bounce it rises to 75% of its previous height.



a) Write the first term and the common ratio of the geometric sequence.

b) Write the general term for the sequence in part a).

c) What height does the ball reach after the 6th bounce?

d) After how many bounces will the ball reach a height of approximately 40 cm?

10. The colour of some clothing fades over time when washed. Suppose a pair of jeans fades by 5% with each washing.

a) What percent of the colour remains after one washing?

b) If $t_1 = 100$, what are the first four terms of the sequence?

c) What is the value of r for your geometric sequence?

d) What percent of the colour remains after 10 washings?

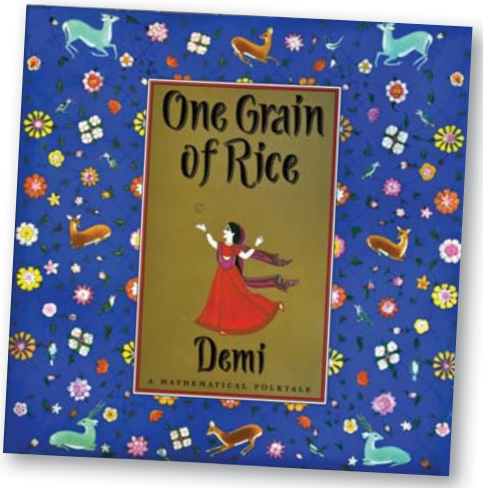
e) How many washings would it take so that only 25% of the original colour remains in the jeans? What assumptions did you make?

11. Pincher Creek, in the foothills of the Rocky Mountains in southern Alberta, is an ideal location to harness the wind power of the chinook winds that blow through the mountain passes. Kinetic energy from the moving air is converted to electricity by wind turbines. In 2004, the turbines generated 326 MW of wind energy, and it is projected that the amount will be 10 000 MW per year by 2010. If this growth were modelled by a geometric sequence, determine the value of the annual growth rate from 2004 to 2010.

Did You Know?

In an average year, a single 660-kW wind turbine produces 2000 MW of electricity, enough power for over 250 Canadian homes. Using wind to produce electricity rather than burning coal will leave 900 000 kg of coal in the ground and emit 2000 tonnes fewer greenhouse gases annually. This has the same positive impact as taking 417 cars off the road or planting 10 000 trees.

12. The following excerpt is taken from the book *One Grain of Rice* by Demi.



Long ago in India, there lived a raja who believed that he was wise and fair. But every year he kept nearly all of the people's rice for himself. Then when famine came, the raja refused to share the rice, and the people went hungry. Then a village girl named Rani devises a clever plan. She does a good deed for the raja, and in return, the raja lets her choose her reward. Rani asks for just one grain of rice, doubled every day for thirty days.

a) Write the sequence of terms for the first five days that Rani would receive the rice.

b) Write the general term that relates the number of grains of rice to the number of days.

c) Use the general term to determine the number of grains of rice that Rani would receive on the 30th day.

13. The Franco-Manitoban community of St-Pierre-Jolys celebrates Les Folies Grenouilles annually in August. Some of the featured activities include a slow pitch tournament, a parade, fireworks, and the Canadian National Frog Jumping Championships. During the competition, competitor's frogs have five chances to reach their maximum jump. One year, a frog by the name of Georges, achieved the winning jump in his 5th try. Georges' first jump was 191.41 cm, his second jump was 197.34 cm, and his third was 203.46 cm. The pattern of Georges' jumps approximated a geometric sequence.

a) By what ratio did Georges improve his performance with each jump? Express your answer to three decimal places.

b) How far was Georges' winning jump? Express your answer to the nearest tenth of a centimetre.

c) The world record frog jump is held by a frog named Santjie of South Africa. Santjie jumped approximately 10.2 m. If Georges, from St-Pierre-Jolys, had continued to increase his jumps following this same geometric sequence, how many jumps would Georges have needed to complete to beat Santjie's world record jump?

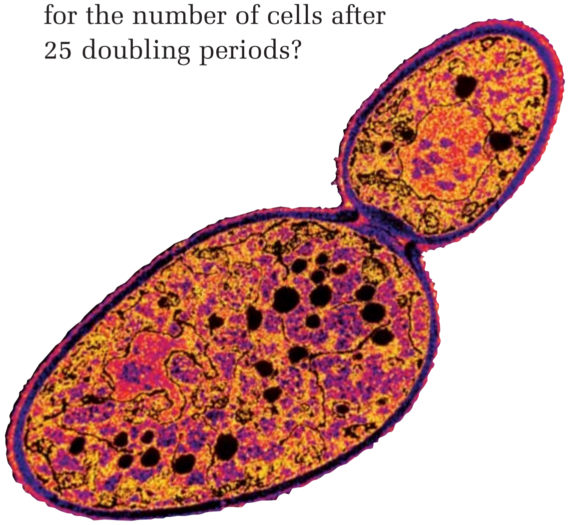
14. Bread and bread products have been part of our diet for centuries. To help bread rise, yeast is added to the dough. Yeast is a living unicellular micro-organism about one hundredth of a millimetre in size. Yeast multiplies by a biochemical process called budding. After mitosis and cell division, one cell results in two cells with exactly the same characteristics.

a) Write a sequence for the first six terms that describes the cell growth of yeast, beginning with a single cell.

b) Write the general term for the growth of yeast.

c) How many cells would there be after 25 doublings?

d) What assumptions would you make for the number of cells after 25 doubling periods?



15. The Arctic Winter Games is a high profile sports competition for northern and arctic athletes. The premier sports are the Dene and Inuit games, which include the arm pull, the one foot high kick, the two foot high kick, and the Dene hand games. The games are held every two years. The first Arctic Winter Games, held in 1970, drew 700 competitors. In 2008, the games were held in Yellowknife and drew 2000 competitors. If the number of competitors grew geometrically from 1970 to 2008, determine the annual rate of growth in the number of competitors from one Arctic Winter Games to the next. Express your answer to the nearest tenth of a percent.

16. Jason Annahatak entered the Russian sledge jump competition at the Arctic Winter Games, held in Yellowknife. Suppose that to prepare for this event, Jason started training by jumping 2 sledges each day for the first week, 4 sledges each day for the second week, 8 sledges each day for the third week, and so on. During the competition, Jason jumped 142 sledges. Assuming he continued his training pattern, how many weeks did it take him to reach his competition number of 142 sledges?

Did You Know?

Sledge jump starts from a standing position. The athlete jumps consecutively over 10 sledges placed in a row, turns around using one jumping movement, and then jumps back over the 10 sledges. This process is repeated until the athlete misses a jump or touches a sledge.

17. At Galaxyland in the West Edmonton Mall, a boat swing ride has been modelled after a basic pendulum design. When the boat first reaches the top of the swing, this is considered to be the beginning of the first swing. A swing is completed when the boat changes direction. On each successive completed swing, the boat travels 96% as far as on the previous swing. The ride finishes when the arc length through which the boat travels is 30 m. If it takes 20 swings for the boat to reach this arc length, determine the arc length through which the boat travels on the first swing. Express your answer to the nearest tenth of a metre.

18. The Russian nesting doll or Matryoshka had its beginnings in 1890. The dolls are made so that the smallest doll fits inside a larger one, which fits inside a larger one, and so on, until all the dolls are hidden inside the largest doll. In a set of 50 dolls, the tallest doll is 60 cm and the smallest is 1 cm. If the decrease in doll size approximates a geometric sequence, determine the common ratio. Express your answer to three decimal places.

19. The primary function for our kidneys is to filter our blood to remove any impurities. Doctors take this into account when prescribing the dosage and frequency of medicine. A person's kidneys filter out 18% of a particular medicine every two hours.

a) How much of the medicine remains after 12 h if the initial dosage was 250 mL? Express your answer to the nearest tenth of a millilitre.

b) When there is less than 20 mL left in the body, the medicine becomes ineffective and another dosage is needed. After how many hours would this happen?

Did You Know?

Every day, a person's kidneys process about 190 L of blood to remove about 1.9 L of waste products and extra water.

20. The charge in a car battery, when the car is left to sit, decreases by about 2% per day and can be modelled by the formula $C = 100(0.98)^d$, where d is the time, in days, and C is the approximate level of charge, as a percent.

a) Copy and complete the chart to show the percent of charge remaining in relation to the time passed.

Time, d (days)	Charge Level, C (%)
0	100
1	
2	
3	

b) Write the general term of this geometric sequence.

c) Explain how this formula is different from the formula $C = 100(0.98)^d$.

d) How much charge is left after 10 days?

21. A coiled basket is made using dried pine needles and sinew. The basket is started from the centre using a small twist and spirals outward and upward to shape the basket. The circular coiling of the basket approximates a geometric sequence, where the radius of the first coil is 6 mm.

a) If the ratio of consecutive coils is 1.22, calculate the radius for the 8th coil.

b) If there are 18 coils, what is the circumference of the top coil of the basket?

Key Ideas

- A geometric series is the expression for the sum of the terms of a geometric sequence.
  For example, $5 + 10 + 20 + 40 + \cdots$ is a geometric series.
- The general formula for the sum of the first n terms of a geometric series with the first term, t_1, and the common ratio, r, is
  $$S_n = \frac{t_1(r^n - 1)}{r - 1},\ r \neq 1$$
- A variation of this formula may be used when the first term, t_1, the common ratio, r, and the nth term, t_n, are known, but the number of terms, n, is not known.
  $$S_n = \frac{rt_n - t_1}{r - 1},\ r \neq 1$$

Check Your Understanding

Practise

1. Determine whether each series is geometric. Justify your answer.

a) $4 + 24 + 144 + 864 + \cdots$

b) $-40 + 20 - 10 + 5 - \cdots$

c) $3 + 9 + 18 + 54 + \cdots$

d) $10 + 11 + 12.1 + 13.31 + \cdots$

2. For each geometric series, state the values of t_1 and r. Then determine each indicated sum. Express your answers as exact values in fraction form and to the nearest hundredth.

a) $6 + 9 + 13.5 + \cdots\ (S_{10})$

b) $18 - 9 + 4.5 + \cdots\ (S_{12})$

c) $2.1 + 4.2 + 8.4 + \cdots\ (S_9)$

d) $0.3 + 0.003 + 0.000\,03 + \cdots\ (S_{12})$

3. What is S_n for each geometric series described? Express your answers as exact values in fraction form.

a) $t_1 = 12,\ r = 2,\ n = 10$

b) $t_1 = 27,\ r = \frac{1}{3},\ n = 8$

c) $t_1 = \frac{1}{256},\ r = -4,\ n = 10$

d) $t_1 = 72,\ r = \frac{1}{2},\ n = 12$

4. Determine S_n for each geometric series. Express your answers to the nearest hundredth, if necessary.

a) $27 + 9 + 3 + \cdots + \frac{1}{243}$

b) $\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \cdots + \frac{128}{6561}$

c) $t_1 = 5,\ t_n = 81\,920,\ r = 4$

d) $t_1 = 3,\ t_n = 46\,875,\ r = -5$

5. What is the value of the first term for each geometric series described? Express your answers to the nearest tenth, if necessary.

   a) $S_n = 33$, $t_n = 48$, $r = -2$

   b) $S_n = 443$, $n = 6$, $r = \frac{1}{3}$

6. The sum of $4 + 12 + 36 + 108 + \cdots + t_n$ is 4372. How many terms are in the series?

7. The common ratio of a geometric series is $\frac{1}{3}$ and the sum of the first 5 terms is 121.

   a) What is the value of the first term?

   b) Write the first 5 terms of the series.

8. What is the second term of a geometric series in which the third term is $\frac{9}{4}$ and the sixth term is $-\frac{16}{81}$? Determine the sum of the first 6 terms. Express your answer to the nearest tenth.

Apply

9. A fan-out system is used to contact a large group of people. The person in charge of the contact committee relays the information to four people. Each of these four people notifies four more people, who in turn each notify four more people, and so on.

   a) Write the corresponding series for the number of people contacted.

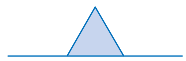
   b) How many people are notified after 10 levels of this system?

10. A tennis ball dropped from a height of 20 m bounces to 40% of its previous height on each bounce. The total vertical distance travelled is made up of upward bounces and downward drops. Draw a diagram to represent this situation. What is the total vertical distance the ball has travelled when it hits the floor for the sixth time? Express your answer to the nearest tenth of a metre.

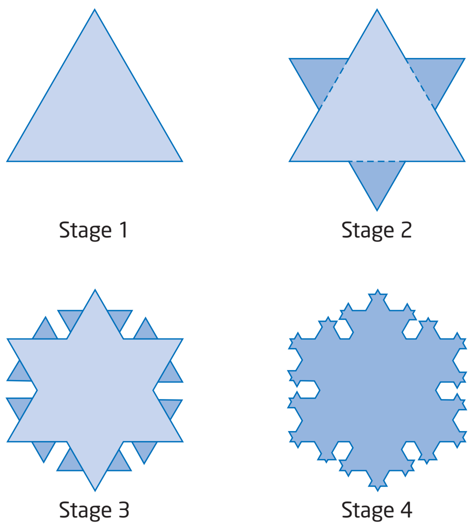
11. Celia is training to run a marathon. In the first week she runs 25 km and increases this distance by 10% each week. This situation may be modelled by the series $25 + 25(1.1) + 25(1.1)^2 + \cdots$. She wishes to continue this pattern for 15 weeks. How far will she have run in total when she completes the 15th week? Express your answer to the nearest tenth of a kilometre.

12. MINI LAB Building the Koch snowflake is a step-by-step process.

   - Start with an equilateral triangle. (Stage 1)
   - In the middle of each line segment forming the sides of the triangle, construct an equilateral triangle with side length equal to $\frac{1}{3}$ of the length of the line segment.

   - Delete the base of this new triangle. (Stage 2)
   - For each line segment in Stage 2, construct an equilateral triangle, deleting its base. (Stage 3)
   - Repeat this process for each line segment, as you move from one stage to the next.



15. When doctors prescribe medicine at equally spaced time intervals, they are aware that the body metabolizes the drug gradually. After some period of time, only a certain percent of the original amount remains. After each dose, the amount of the drug in the body is equal to the amount of the given dose plus the amount remaining from the previous doses. The amount of the drug present in the body after the nth dose is modelled by a geometric series where t_1 is the prescribed dosage and r is the previous dose remaining in the body.

    Suppose a person with an ear infection takes a 200-mg ampicillin tablet every 4 h. About 12% of the drug in the body at the start of a four-hour period is still present at the end of that period. What amount of ampicillin is in the body, to the nearest tenth of a milligram,

    a) after taking the third tablet?

    b) after taking the sixth tablet?

Extend

16. Determine the number of terms, n, if $3 + 3^2 + 3^3 + \cdots + 3^n = 9840$.

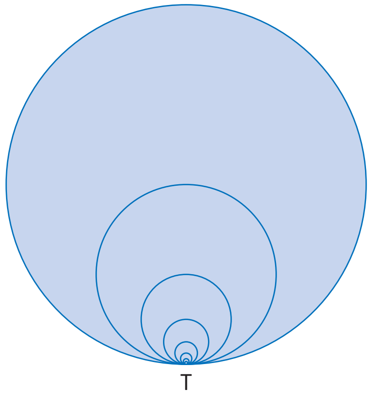
17. The third term of a geometric series is 24 and the fourth term is 36. Determine the sum of the first 10 terms. Express your answer as an exact fraction.

18. Three numbers, a, b, and c, form a geometric series so that $a + b + c = 35$ and $abc = 1000$. What are the values of a, b, and c?

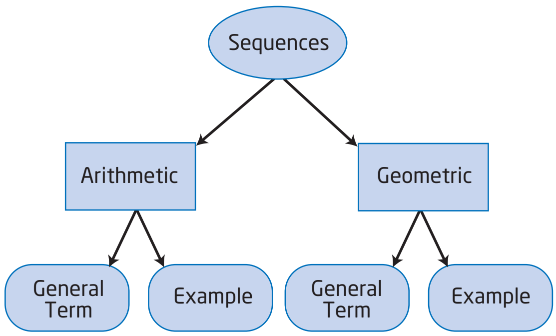
19. The sum of the first 7 terms of a geometric series is 89, and the sum of the first 8 terms is 104. What is the value of the eighth term?

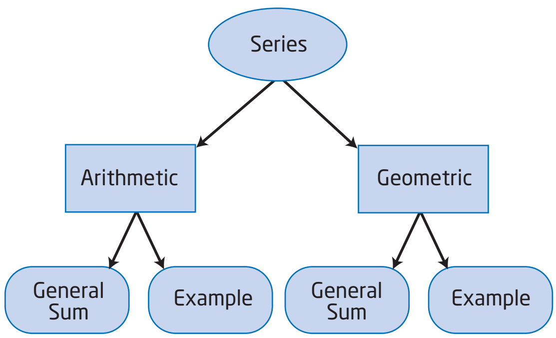
Create Connections

20. A fractal is created as follows: A circle is drawn with radius 8 cm. Another circle is drawn with half the radius of the previous circle. The new circle is tangent to the previous circle at point T as shown. Suppose this pattern continues through five steps. What is the sum of the areas of the circles? Express your answer as an exact fraction.



21. Copy the following flowcharts. In the appropriate segment of each chart, give a definition, a general term or sum, or an example, as required.





Key Ideas

- An infinite geometric series is a geometric series that has an infinite number of terms; that is, the series has no last term.
- An infinite series is said to be convergent if its sequence of partial sums approaches a finite number. This number is the sum of the infinite series. An infinite series that is not convergent is said to be divergent.
- An infinite geometric series has a sum when $-1 < r < 1$ and the sum is given by $S_\infty = \frac{t_1}{1 - r}$.

Check Your Understanding

Practise

1. State whether each infinite geometric series is convergent or divergent.

a) $t_1 = -3, r = 4$

b) $t_1 = 4, r = -\frac{1}{4}$

c) $125 + 25 + 5 + \cdots$

d) $(-2) + (-4) + (-8) + \cdots$

e) $\frac{243}{3125} - \frac{81}{625} + \frac{27}{25} - \frac{9}{5} + \cdots$

2. Determine the sum of each infinite geometric series, if it exists.

a) $t_1 = 8, r = -\frac{1}{4}$

b) $t_1 = 3, r = \frac{4}{3}$

c) $t_1 = 5, r = 1$

d) $1 + 0.5 + 0.25 + \cdots$

e) $4 - \frac{12}{5} + \frac{36}{25} - \frac{108}{125} + \cdots$

3. Express each of the following as an infinite geometric series. Determine the sum of the series.

a) $0.\overline{87}$

b) $0.4\overline{37}$

4. Does $0.999... = 1$? Support your answer.

5. What is the sum of each infinite geometric series?

a) $5 + 5\left(\frac{2}{3}\right) + 5\left(\frac{2}{3}\right)^2 + 5\left(\frac{2}{3}\right)^3 + \cdots$

b) $1 + \left(-\frac{1}{4}\right) + \left(-\frac{1}{4}\right)^2 + \left(-\frac{1}{4}\right)^3 + \cdots$

c) $7 + 7\left(\frac{1}{2}\right) + 7\left(\frac{1}{2}\right)^2 + 7\left(\frac{1}{2}\right)^3 + \cdots$

Apply

6. The sum of an infinite geometric series is 81, and its common ratio is $\frac{2}{3}$. What is the value of the first term? Write the first three terms of the series.

7. The first term of an infinite geometric series is -8, and its sum is $-\frac{40}{3}$. What is the common ratio? Write the first four terms of the series.

8. In its first month, an oil well near Virden, Manitoba produced 24 000 barrels of crude. Every month after that, it produced 94% of the previous month's production.

a) If this trend continued, what would be the lifetime production of this well?

b) What assumption are you making? Is your assumption reasonable?

9. The infinite series given by $1 + 3x + 9x^2 + 27x^3 + \cdots$ has a sum of 4. What is the value of x? List the first four terms of the series.

10. The sum of an infinite series is twice its first term. Determine the value of the common ratio.

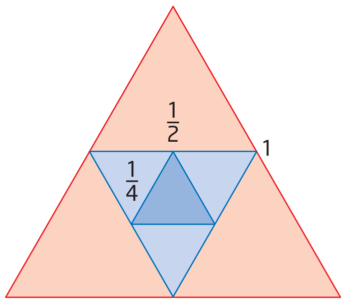
11. Each of the following represents an infinite geometric series. For what values of x will each series be convergent?

a) $5 + 5x + 5x^2 + 5x^3 + \cdots$

b) $1 + \frac{x}{3} + \frac{x^2}{9} + \frac{x^3}{27} + \cdots$

c) $2 + 4x + 8x^2 + 16x^3 + \cdots$

12. Each side of an equilateral triangle has length of 1 cm. The midpoints of the sides are joined to form an inscribed equilateral triangle. Then, the midpoints of the sides of that triangle are joined to form another triangle. If this process continues forever, what is the sum of the perimeters of the triangles?



13. The length of the initial swing of a pendulum is 50 cm. Each successive swing is 0.8 times the length of the previous swing. If this process continues forever, how far will the pendulum swing?

14. Andrew uses the formula for the sum of an infinite geometric series to evaluate $1 + 1.1 + 1.21 + 1.331 + \cdots$. He calculates the sum of the series to be 10. Is Andrew's answer reasonable? Explain.

15. A ball is dropped from a height of 16 m. The ball rebounds to one half of its previous height each time it bounces. If the ball keeps bouncing, what is the total vertical distance the ball travels?

16. A pile driver pounds a metal post into the ground. With the first impact, the post moves 30 cm; with the second impact it moves 27 cm. Predict the total distance that the post will be driven into the ground if

a) the distances form a geometric sequence and the post is pounded 8 times

b) the distances form a geometric sequence and the post is pounded indefinitely

17. Dominique and Rita are discussing the series $-\frac{1}{3} + \frac{4}{9} - \frac{16}{27} + \cdots$. Dominique says that the sum of the series is $-\frac{1}{7}$. Rita says that the series is divergent and has no sum.

a) Who is correct?

b) Explain your reasoning.

18. A hot air balloon rises 25 m in its first minute of flight. Suppose that in each succeeding minute the balloon rises only 80% as high as in the previous minute. What would be the balloon's maximum altitude?

Hot air balloon rising over Calgary.

Extend

19. A square piece of paper with a side length of 24 cm is cut into four small squares, each with side lengths of 12 cm. Three of these squares are placed side by side. The remaining square is cut into four smaller squares, each with side lengths of 6 cm. Three of these squares are placed side by side with the bigger squares. The fourth square is cut into four smaller squares and three of these squares are placed side by side with the bigger squares. Suppose this process continues indefinitely. What is the length of the arrangement of squares?

20. The sum of the series
$0.98 + 0.98^2 + 0.98^3 + \cdots + 0.98^n = 49$.
The sum of the series
$0.02 + 0.0004 + 0.000\ 008 + \cdots = \frac{1}{49}$.
The common ratio in the first series is 0.98 and the common ratio in the second series is 0.02. The sum of these ratios is equal to 1. Suppose that $\frac{1}{z} = x + x^2 + x^3 + \cdots$, where z is an integer and $x = \frac{1}{z + 1}$.

a) Create another pair of series that would follow this pattern, where the sum of the common ratios of the two series is 1.

b) Determine the sum of each series using the formula for the sum of an infinite series.

Create Connections

21. Under what circumstances will an infinite geometric series converge?

22. The first two terms of a series are 1 and $\frac{1}{4}$. Determine a formula for the sum of n terms if the series is

a) an arithmetic series

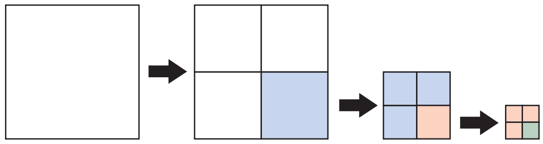
b) a geometric series

c) an infinite geometric series

23. MINI LAB Work in a group of three.

Step 1 Begin with a large sheet of grid paper and draw a square. Assume that the area of this square is 1.

Step 2 Cut the square into 4 equal parts. Distribute one part to each member of your group. Cut the remaining part into 4 equal parts. Again distribute one part to each group member. Subdivide the remaining part into 4 equal parts. Suppose you could continue this pattern indefinitely.



Step 3 Write a sequence for the fraction of the original square that each student received at each stage.

n	1	2	3	4
Fraction of Paper				

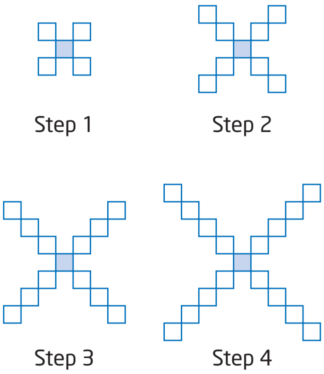
Step 4 Write the total area of paper each student has as a series of partial sums. What do you expect the sum to be?

Project Corner Petroleum

- The Athabasca Oil Sands have estimated oil reserves in excess of that of the rest of the world. These reserves are estimated to be 1.6 trillion barrels.
- Canada is the seventh largest oil producing country in the world. In 2008, Canada produced an average of 438 000 m^3 per day of crude oil, crude bitumen, and natural gas.
- As Alberta's reserves of light crude oil began to deplete, so did production. By 1997, Alberta's light crude oil production totalled 37.3 million cubic metres. This production has continued to decline each year since, falling to just over half of its 1990 total at 21.7 million cubic metres in 2005.

c) How many days would you need to contact the 625 people in your neighbourhood?

10. A new set of designs is created by the addition of squares to the previous pattern.



a) Determine the total number of squares in the 15th step of this design.

b) Determine the total number of squares required to build all 15 steps.

11. A concert hall has 10 seats in the first row. The second row has 12 seats. If each row has 2 seats more than the row before it and there are 30 rows of seats, how many seats are in the entire concert hall?

1.3 Geometric Sequences, pages 32–45

12. Determine whether each of the following sequences is geometric. If it is geometric, determine the common ratio, r, the first term, t_1, and the general term of the sequence.

a) 3, 6, 10, 15, …

b) 1, −2, 4, −8, …

c) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

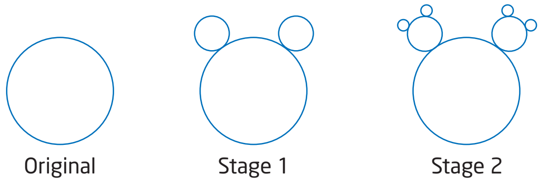
d) $\frac{16}{9}, -\frac{3}{4}, 1, \ldots$

13. A culture initially has 5000 bacteria, and the number increases by 8% every hour.

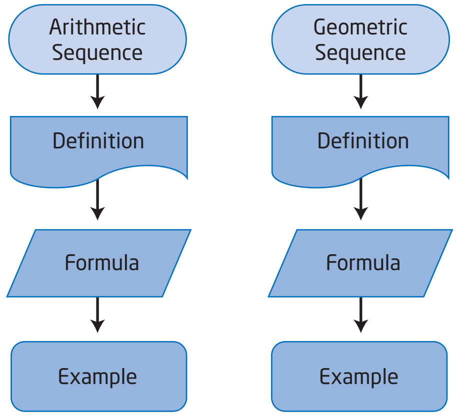
a) How many bacteria are present at the end of 5 h?

b) Determine a formula for the number of bacteria present after n hours.

14. In the Mickey Mouse fractal shown below, the original diagram has a radius of 81 cm. Each successive circle has a radius $\frac{1}{3}$ of the previous radius. What is the circumference of the smallest circle in the 4th stage?



15. Use the following flowcharts to describe what you know about arithmetic and geometric sequences.



1.4 Geometric Series, pages 46–57

16. Decide whether each of the following statements relates to an arithmetic series or a geometric series.

a) A sum of terms in which the difference between consecutive terms is constant.

b) A sum of terms in which the ratio of consecutive terms is constant.

c) $S_n = \frac{t_1(r^n - 1)}{r - 1}, r \neq 1$

d) $S_n = \frac{n[2t_1 + (n - 1)d]}{2}$

e) $\frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1 + \cdots$

f) $\frac{1}{4} + \frac{1}{6} + \frac{1}{9} + \frac{2}{27} + \cdots$

17. Determine the sum indicated for each of the following geometric series.

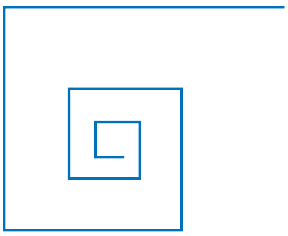
a) $6 + 9 + 13.5 + \cdots\ (S_{10})$

b) $18 + 9 + 4.5 + \cdots\ (S_{12})$

c) $6000 + 600 + 60 + \cdots\ (S_{20})$

d) $80 + 20 + 5 + \cdots\ (S_9)$

18. A student programs a computer to draw a series of straight lines with each line beginning at the end of the previous line and at right angles to it. The first line is 4 mm long. Each subsequent line is 25% longer than the previous one, so that a spiral shape is formed as shown.



a) What is the length, in millimetres, of the eighth straight line drawn by the program? Express your answer to the nearest tenth of a millimetre.

b) Determine the total length of the spiral, in metres, when 20 straight lines have been drawn. Express your answer to the nearest hundredth of a metre.

1.5 Infinite Geometric Series, pages 58–65

19. Determine the sum of each of the following infinite geometric series.

a) $5 + 5\left(\frac{2}{3}\right) + 5\left(\frac{2}{3}\right)^2 + 5\left(\frac{2}{3}\right)^3 + \cdots$

b) $1 + \left(-\frac{1}{3}\right) + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^3 + \cdots$

20. For each of the following series, state whether it is convergent or divergent. For those that are convergent, determine the sum.

a) $8 + 4 + 2 + 1 + \cdots$

b) $8 + 12 + 27 + 40.5 + \cdots$

c) $-42 + 21 - 10.5 + 5.25 - \cdots$

d) $\frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{32} + \cdots$

21. Given the infinite geometric series:
$7 - 2.8 + 1.12 - 0.448 + \cdots$

a) What is the common ratio, r?

b) Determine S_1, S_2, S_3, S_4, and S_5.

c) What is the particular value that the sums are approaching?

d) What is the sum of the series?

22. Draw four squares adjacent to each other. The first square has a side length of 1 unit, the second has a side length of $\frac{1}{2}$ unit, the third has a side length of $\frac{1}{4}$ unit, and the fourth has a side length of $\frac{1}{8}$ unit.

a) Calculate the area of each square. Do the areas form a geometric sequence? Justify your answer.

b) What is the total area of the four squares?

c) If the process of adding squares with half the side length of the previous square continued indefinitely, what would the total area of all the squares be?

23. a) Copy and complete each of the following statements.

- A series is geometric if there is a common ratio r such that ______.
- An infinite geometric series converges if ______.
- An infinite geometric series diverges if ______.

b) Give two examples of convergent infinite geometric series one with positive common ratio and one with negative common ratio. Determine the sum of each of your series.