Trigonometry – Open-ended Solutions

1. **a)** II **b)** III **c)** IV **d)** I

e) $612^\circ - 360^\circ = 252^\circ$, III **f)** $-537^\circ + 2 \times 360^\circ = 183^\circ$, III **g)** $1100^\circ - 3 \times 360^\circ = 20^\circ$, I

h) $\frac{6325^\circ}{360^\circ} = 17^+$, $6325^\circ - 17 \times 360^\circ = 205^\circ$, III **i)** $810^\circ - 2 \times 360^\circ = 90^\circ$, not in quadrant

j) $-900^\circ + 3 \times 360^\circ = 180^\circ$, not in quadrant

2. **a)** $150^\circ + 360^\circ = 510^\circ$, $150^\circ - 360^\circ = -210^\circ$ **b)** $-150^\circ + 360^\circ = 210^\circ$, $-150^\circ - 360^\circ = -510^\circ$

c) $314^\circ + 360^\circ = 674^\circ$, $314^\circ - 360^\circ = -46^\circ$ **d)** $-314^\circ + 360^\circ = 46^\circ$, $-314^\circ - 360^\circ = -674^\circ$

e) $612^\circ - 360^\circ = 252^\circ$, $612^\circ - 2 \times 360^\circ = -108^\circ$ **f)** $-537^\circ + 2 \times 360^\circ = 183^\circ$, $-537^\circ + 360^\circ = -177^\circ$

g) $1100^\circ - 360^\circ = 740^\circ$, $1100^\circ - 4 \times 360^\circ = -340^\circ$ **h)** $6325^\circ - 360^\circ = 5965^\circ$, $6325^\circ - 18 \times 360^\circ = -155^\circ$

i) $810^\circ - 2 \times 360^\circ = 90^\circ$, $810^\circ - 3 \times 360^\circ = -270^\circ$ **j)** $-900^\circ + 3 \times 360^\circ = 180^\circ$, $-900^\circ + 360^\circ = -540^\circ$

Note: *There are an infinite number of possible answers for each question. List above represents just one possible answer for each question.*

3. **a)** 30° **b)** 30° **c)** 46° **d)** 46° **e)** 72°

f) 3° **g)** 20° **h)** 25° **i)** 90° **j)** 0°

4. **a)** $45^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{4}$ **b)** $90^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{2}$ **c)** $150^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{6}$ **d)** $240^\circ \times \frac{\pi}{180^\circ} = \frac{4\pi}{3}$

e) $300^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{3}$ **f)** $360^\circ \times \frac{\pi}{180^\circ} = 2\pi$ **g)** $405^\circ \times \frac{\pi}{180^\circ} = \frac{9\pi}{4}$ **h)** $420^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{3}$

i) $450^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{2}$ **j)** $630^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{2}$

5. **a)** $70^\circ \times \frac{\pi}{180^\circ} = 1.222$ **b)** $37.5^\circ \times \frac{\pi}{180^\circ} = 0.654$ **c)** $130^\circ \times \frac{\pi}{180^\circ} = 2.269$ **d)** $\frac{90^\circ}{\pi} \times \frac{\pi}{180^\circ} = 0.5$

e) $400^\circ \times \frac{\pi}{180^\circ} = 6.981$ **f)** $527^\circ \times \frac{\pi}{180^\circ} = 9.198$ **g)** $-248^\circ \times \frac{\pi}{180^\circ} = -4.328$

h) $718^\circ \times \frac{\pi}{180^\circ} = 12.531$ **i)** $1025^\circ \times \frac{\pi}{180^\circ} = 17.890$ **j)** $-1349^\circ \times \frac{\pi}{180^\circ} = -23.544$

6. **a)** $\frac{\pi}{3} \times \frac{180^\circ}{\pi} = 60^\circ$ **b)** $\frac{5\pi}{6} \times \frac{180^\circ}{\pi} = 150^\circ$ **c)** $\frac{3\pi}{4} \times \frac{180^\circ}{\pi} = 135^\circ$ **d)** $\frac{11\pi}{6} \times \frac{180^\circ}{\pi} = 330^\circ$

e) $\frac{17\pi}{6} \times \frac{180^\circ}{\pi} = 510^\circ$ **f)** $\frac{21\pi}{4} \times \frac{180^\circ}{\pi} = 945^\circ$ **g)** $\frac{11\pi}{3} \times \frac{180^\circ}{\pi} = 660^\circ$ **h)** $\frac{20\pi}{3} \times \frac{180^\circ}{\pi} = 1200^\circ$

i) $\frac{31\pi}{6} \times \frac{180^\circ}{\pi} = 930^\circ$ **j)** $\frac{23\pi}{4} \times \frac{180^\circ}{\pi} = 1035^\circ$

7. a) $3 \times \frac{180^\circ}{\pi} = 171.9^\circ$ b) $-4 \times \frac{180^\circ}{\pi} = -229.2^\circ$ c) $2.7 \times \frac{180^\circ}{\pi} = 154.7^\circ$

d) $-1.2 \times \frac{180^\circ}{\pi} = -68.8^\circ$ e) $8.2 \times \frac{180^\circ}{\pi} = 469.8^\circ$ f) $-12.8 \times \frac{180^\circ}{\pi} = -733.4^\circ$

8. a) $a = r\theta \;\rightarrow\; 3 = r \cdot 30^\circ \cdot \frac{\pi}{180} \;\rightarrow\; 3 = r \cdot \frac{\pi}{6} \;\rightarrow\; r = \frac{18}{\pi} \;\rightarrow\; r \approx 5.73 \text{ cm}$

b) $a = r\theta \;\rightarrow\; a = 15 \cdot 130^\circ \cdot \frac{\pi}{180} \;\rightarrow\; a = \frac{65\pi}{6} \;\rightarrow\; a \approx 34.03 \text{ cm}$

c) $a = r\theta \;\rightarrow\; 5 = 6 \cdot \theta \cdot \frac{\pi}{180^\circ} \;\rightarrow\; 5 = \frac{\pi}{30^\circ} \cdot \theta \;\rightarrow\; \theta = \frac{150^\circ}{\pi} \rightarrow \; \theta \approx 47.75^\circ$

d) (*i*) The minute hand makes one-half of a turn in one half an hour so the radian value is π.

(*ii*) The hour hand makes one twenty-fourth of a turn in one half hour so the radian value is $\frac{1}{24} \cdot 2\pi = \frac{\pi}{12}$

9. a) $r = \sqrt{4^2 + 3^2} = 5$, $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$, $\cot\theta = \frac{4}{3}$, $\sec\theta = \frac{5}{4}$, $\csc\theta = \frac{5}{3}$

b) $r = \sqrt{\left(-\sqrt{7}\right)^2 + 3^2} = 4$, $\sin\theta = \frac{3}{4}$, $\cos\theta = \frac{-\sqrt{7}}{4}$, $\tan\theta = \frac{-3}{\sqrt{7}}$, $\cot\theta = \frac{-\sqrt{7}}{3}$, $\sec\theta = \frac{-4}{\sqrt{7}}$, $\csc\theta = \frac{4}{3}$

c) $r = \sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2} = 2$, $\sin\theta = \frac{-1}{2}$, $\cos\theta = \frac{-\sqrt{3}}{2}$, $\tan\theta = \frac{1}{\sqrt{3}}$, $\cot\theta = \sqrt{3}$, $\sec\theta = \frac{-2}{\sqrt{3}}$, $\csc\theta = -2$

d) $r = \sqrt{\left(\sqrt{5}\right)^2 + (-2)^2} = 3$, $\sin\theta = \frac{-2}{3}$, $\cos\theta = \frac{\sqrt{5}}{3}$, $\tan\theta = \frac{-2}{\sqrt{5}}$, $\cot\theta = \frac{-\sqrt{5}}{2}$, $\sec\theta = \frac{3}{\sqrt{5}}$, $\csc\theta = \frac{-3}{2}$

10. a) $(\sec x \cdot \csc x - \cot x)(\sin x - \csc x) =$

$$\left(\frac{1}{\cos x} \cdot \frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)\left(\sin x - \frac{1}{\sin x}\right) = \left(\frac{1}{\cos x} \cdot \frac{1}{\sin x} - \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x}\right)\left(\sin x - \frac{1}{\sin x}\right) =$$

$$\left(\frac{1 - \cos^2 x}{\sin x \cos x}\right)\left(\frac{\sin^2 x - 1}{\sin x}\right) = \left(\frac{\sin^2 x}{\sin x \cos x}\right)\left(-\frac{\cos^2 x}{\sin x}\right) = -\cos x$$

b) $$\frac{\frac{\cot x + 1}{\cot x} - 1}{\frac{\cot x - 1}{\cot x} - 1} = \frac{\frac{\cot x + 1 - \cot x}{\cot x}}{\frac{\cot x - 1 - \cot x}{\cot x}} = \frac{1}{-1} = -1$$

c) $$\frac{\tan^2 x}{\cos^2 x + \sin^2 x + \tan^2 x} = \frac{\tan^2 x}{1 + \tan^2 x} = \frac{\tan^2 x}{\sec^2 x} = \frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x = \sin^2 x$$

d) $$\frac{\cos x \cdot \tan x + \sin x}{2\tan x} = \frac{\cos x}{2\sin x}\left(\cos x \cdot \frac{\sin x}{\cos x} + \sin x\right) = \frac{\cos x}{2\sin x} \cdot 2\sin x = \cos x$$

10. e) $\dfrac{1-\sec^2 x}{\sec^2 x}-\cos^2 x=\cos^2 x\left(-\tan^2 x\right)-\cos^2 x=\cos^2 x\left(\dfrac{-\sin^2 x}{\cos^2 x}\right)-\cos^2 x=$

$-\sin^2 x-\cos^2 x=-\left(\sin^2 x+\cos^2 x\right)=-1$

f) $$\frac{\sec x-\cos x}{\csc x-\sin x}=\frac{\dfrac{1}{\cos x}-\cos x}{\dfrac{1}{\sin x}-\sin x}=\frac{\dfrac{1-\cos^2 x}{\cos x}}{\dfrac{1-\sin^2 x}{\sin x}}=\frac{(1-\cos^2 x)\sin x}{(1-\sin^2 x)\cos x}=\frac{(\sin^2 x)\sin x}{(\cos^2 x)\cos x}=\frac{\sin^3 x}{\cos^3 x}=\tan^3 x$$

g) $$\frac{\cot x(\sin x+\tan x)}{\csc x+\cot x}=\frac{\dfrac{\cos x}{\sin x}\left(\sin x+\dfrac{\sin x}{\cos x}\right)}{\dfrac{1}{\sin x}+\dfrac{\cos x}{\sin x}}=\frac{\dfrac{\cos x}{\sin x}\left(\dfrac{\sin x\cos x+\sin x}{\cos x}\right)}{\dfrac{1+\cos x}{\sin x}}=\frac{\sin x(\cos x+1)}{1+\cos x}=\sin x$$

h) $$\frac{\sec x-\cos x}{\tan x}=\frac{\dfrac{1}{\cos x}-\cos x}{\dfrac{\sin x}{\cos x}}=\frac{\dfrac{1-\cos^2 x}{\cos x}}{\dfrac{\sin x}{\cos x}}=\frac{1-\cos^2 x}{\sin x}=\frac{\sin^2 x}{\sin x}=\sin x$$

i) $$\frac{\sec^2 x\,(1+\csc x)-\tan x\,(\sec x+\tan x)}{\csc x\,(1+\sin x)}=\frac{\dfrac{1}{\cos^2 x}\left(1+\dfrac{1}{\sin x}\right)-\dfrac{\sin x}{\cos x}\left(\dfrac{1}{\cos x}+\dfrac{\sin x}{\cos x}\right)}{\dfrac{1+\sin x}{\sin x}}=$$

$$\frac{\sin x}{1+\sin x}\left[\frac{1}{\cos^2 x}\left(\frac{\sin x+1}{\sin x}\right)-\frac{\sin x}{\cos x}\left(\frac{1+\sin x}{\cos x}\right)\right]=\frac{1}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}=\frac{1-\sin^2 x}{\cos^2 x}=\frac{\cos^2 x}{\cos^2 x}=1$$

j) $$\frac{\csc^2 x+\sec^2 x}{\csc x\sec x}=\frac{\dfrac{1}{\sin^2 x}+\dfrac{1}{\cos^2 x}}{\dfrac{1}{\sin x}\cdot\dfrac{1}{\cos x}}=\frac{\cos^2 x+\sin^2 x}{\sin^2 x\cos^2 x}\cdot\sin x\cos x=\frac{1}{\sin x\cos x}=\csc x\cdot\sec x$$

k) $$\frac{\cos x+\cot x}{1+\csc x}=\frac{\cos x+\dfrac{\cos x}{\sin x}}{1+\dfrac{1}{\sin x}}=\frac{\dfrac{\sin x\cdot\cos x+\cos x}{\sin x}}{\dfrac{\sin x+1}{\sin x}}=\frac{\cos x(\sin x+1)}{\sin x+1}=\cos x$$

l) $$\frac{\sec x}{\tan x-\cot x}=\frac{\dfrac{1}{\cos x}}{\dfrac{\sin x}{\cos x}+\dfrac{\cos x}{\sin x}}=\frac{\dfrac{1}{\cos x}}{\dfrac{\sin^2 x+\cos^2 x}{\sin x\cdot\cos x}}=\frac{\dfrac{1}{\cos x}}{\dfrac{1}{\sin x\cdot\cos x}}=\frac{\sin x\cdot\cos x}{\cos x}=\sin x$$

11. *Note: expression is undefined whenever sin θ or cos θ has a zero value in the denominator.*

a) $\dfrac{\cot\theta}{1+\sin\theta}=\dfrac{\cos\theta}{\sin\theta(1+\sin\theta)}$, therefore, $\sin\theta\neq 0,\,-1$; for $0\le\theta<2\pi$, $\theta\neq 0,\pi,\dfrac{3\pi}{2}$

b) $\dfrac{\sec x}{1-\cos x}=\dfrac{1}{\cos x(1-\cos x)}$, therefore, $\cos x\neq 0,1$; for $0\le x<2\pi$, $x\neq 0,\dfrac{\pi}{2},\dfrac{3\pi}{2}$

11. c) $\dfrac{1}{2\cos^2 x + \cos x - 1} = \dfrac{1}{(2\cos x - 1)(\cos x + 1)}$, therefore, $\cos x \neq \dfrac{1}{2}, -1$;

for $0 \leq x < 2\pi,\ x \neq \dfrac{\pi}{3},\ \pi,\ \dfrac{5\pi}{3}$

d) $\cot x + \tan x = \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}$, therefore, $\sin x \neq 0$, $\cos x \neq 0$

for $0 \leq x < 2\pi,\ x \neq 0,\ \dfrac{\pi}{2},\ \pi,\ \dfrac{3\pi}{2}$

12. a) $\sin\theta > 0$ in quadrant I, II, $\sec\theta > 0$ in quadrant I, IV, therefore, answer is quadrant I.

b) $\tan\theta < 0$ in quadrant II, IV, $\cos\theta > 0$ in quadrant I, IV, therefore, answer is quadrant IV.

c) $\csc\theta > 0$ in quadrant I, II, $\cot\theta < 0$ in quadrant II, IV, therefore, answer is quadrant II.

d) $\cos\theta < 0$ in quadrant II, III, $\csc\theta < 0$ in quadrant III, IV, therefore, answer is quadrant III.

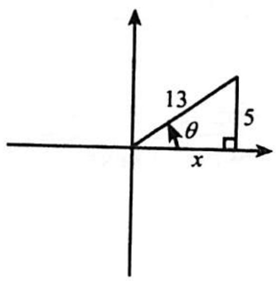
e) $\sin\theta < 0$ in quadrant III, IV, $\tan\theta < 0$ in quadrant II, IV, therefore, answer is quadrant IV.

f) $\cot\theta > 0$ in quadrant I, III, $\sec\theta < 0$ in quadrant II, III, therefore, answer is quadrant III.

g) $\tan\theta < 0$ in quadrant II, IV, $\csc\theta > 0$ in quadrant I, II, therefore, answer is quadrant II.

h) $\cos\theta$ and $\sec\theta$ are reciprocals of each other so it is impossible for one to be negative and the other positive

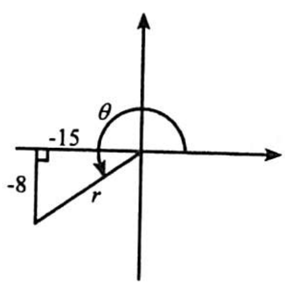
13. a)



$x^2 + 5^2 = 13^2$
$x = 12$

$\cos\theta = \dfrac{12}{13}$, $\tan\theta = \dfrac{5}{12}$, $\cot\theta = \dfrac{12}{5}$, $\sec\theta = \dfrac{13}{12}$, $\csc\theta = \dfrac{13}{5}$

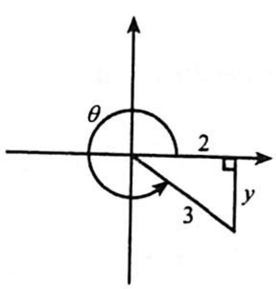
b)



$r^2 = (8)^2 + (-15)^2$
$r = 17$

$\sin\theta = \dfrac{-8}{17}$, $\cos\theta = \dfrac{-15}{17}$, $\cot\theta = \dfrac{15}{8}$, $\sec\theta = \dfrac{-17}{15}$, $\csc\theta = \dfrac{-17}{8}$

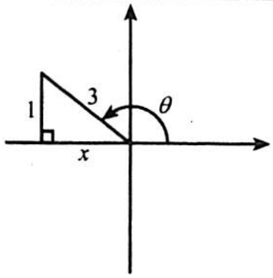
c)



$y^2 + 2^2 = 3^2 \rightarrow y = -\sqrt{5}$ $\sin\theta = \dfrac{-\sqrt{5}}{3}$, $\cos\theta = \dfrac{2}{3}$

$\tan\theta = \dfrac{-\sqrt{5}}{2}$, $\cot\theta = \dfrac{-2}{\sqrt{5}} = \dfrac{-2\sqrt{5}}{5}$, $\csc\theta = \dfrac{-3}{\sqrt{5}} = \dfrac{-3\sqrt{5}}{5}$

13. d)



$\csc x > 0$ in quadrant I, II, $\tan x < 0$ in quadrant II, IV, therefore, answer in quadrant II.

$x^2 + 1^2 = 3^2 \rightarrow x = -2\sqrt{2}$ $\quad \sin\theta = \dfrac{1}{3}, \quad \cos\theta = \dfrac{-2\sqrt{2}}{3}$

$\tan x = \dfrac{-1}{2\sqrt{2}} = \dfrac{-\sqrt{2}}{4}, \quad \cot x = -2\sqrt{2}, \quad \sec\theta = \dfrac{-3}{2\sqrt{2}} = \dfrac{-3\sqrt{2}}{4}$

14. a) $\sin 120^\circ = \sin 60^\circ = \dfrac{\sqrt{3}}{2}$

b) $\cot 135^\circ = -\cot 45^\circ = -1$

c) $\cos 330^\circ = \cos 30^\circ = \dfrac{\sqrt{3}}{2}$

d) $\tan 660^\circ = \tan 300^\circ = -\tan 60^\circ = -\sqrt{3}$

e) $\csc 1125^\circ = \csc 45^\circ = \sqrt{2}$

f) $\sec\dfrac{\pi}{6} = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}$

g) $\sin\dfrac{5\pi}{4} = -\sin\dfrac{\pi}{4} = \dfrac{-1}{\sqrt{2}} = \dfrac{-\sqrt{2}}{2}$

h) $\tan\dfrac{11\pi}{6} = -\tan\dfrac{\pi}{6} = \dfrac{-1}{\sqrt{3}} = \dfrac{-\sqrt{3}}{3}$

i) $\csc\dfrac{19\pi}{6} = \csc\dfrac{7\pi}{6} = -\csc\dfrac{\pi}{6} = \dfrac{2}{-1} = -2$

j) $\cot\dfrac{13\pi}{3} = \cot\dfrac{\pi}{3} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

k) $\cot(-240^\circ) = \cot 120^\circ = -\cot 60^\circ = \dfrac{-\sqrt{3}}{3}$

l) $\sec(-945^\circ) = \sec 135^\circ = -\sec 45^\circ = \dfrac{\sqrt{2}}{-1} = -\sqrt{2}$

m) $\cos\left(\dfrac{-5\pi}{3}\right) = \cos\dfrac{\pi}{3} = \dfrac{1}{2}$

n) $\tan\left(\dfrac{-29\pi}{6}\right) = \tan\dfrac{7\pi}{6} = \tan\dfrac{\pi}{6} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

o) $\sin\left(\dfrac{-20\pi}{3}\right) = \sin\dfrac{4\pi}{3} = -\sin\dfrac{\pi}{3} = \dfrac{-\sqrt{3}}{2}$

p) $\csc\left(\dfrac{-27\pi}{4}\right) = \csc\dfrac{5\pi}{4} = -\csc\dfrac{\pi}{4} = -\dfrac{\sqrt{2}}{1} = -\sqrt{2}$

15. a) $\cos x > 0$ in quadrant I, IV, therefore, by special angles, answers are $\dfrac{\pi}{6}, \dfrac{11\pi}{6}$

b) $\sin x < 0$ in quadrant III, IV, therefore, by special angles, answers are $\dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

c) $\tan x < 0$ in quadrant II, IV, therefore, by special angles, answers are $\dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

d) $\csc x > 0$ in quadrant I, II, therefore, by special angles, answers are $\dfrac{\pi}{6}, \dfrac{5\pi}{6}$

e) $\sec x < 0$ in quadrant II, III, therefore, by special angles, answers are $\dfrac{3\pi}{4}, \dfrac{5\pi}{4}$

f) $\sin x = -1$ by special angle at $\dfrac{3\pi}{2}$

g) $\cot x =$ undefined by special angles at $0, \pi$

h) $\cos x = 0$ by special angles at $\dfrac{\pi}{2}, \dfrac{3\pi}{2}$

i) $\csc x =$ undefined by special angles at $0, \pi$

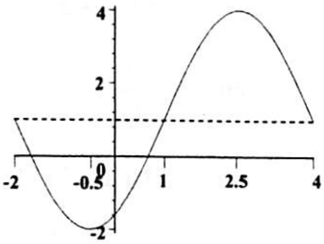
j) $\sec x = -1$ by special angles at π

16. a) Remember the period of $y = a\tan bx$ is $\dfrac{\pi}{|b|}$, so the period of $y = 3\tan\dfrac{1}{3}x$ is $\dfrac{\pi}{\frac{1}{3}} = 3\pi$

b) Remember the period of $y = a\cot bx$ is $\dfrac{\pi}{|b|}$, so period of $y = -2\cot\dfrac{\pi}{2}x$ is $\dfrac{\pi}{\frac{\pi}{2}} = 2$

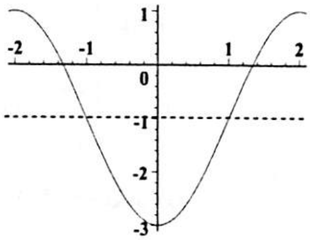
16. c) $y = -3\sin\frac{\pi}{3}(x+2)+1$, amplitude $= |-3| = 3$, period $= \dfrac{2\pi}{\frac{\pi}{3}} = 6$

phase shift $= \frac{\pi}{3}(x+2) = 0 \rightarrow x = -2$, vertical displacement = 1



d) $y = 2\cos(\frac{\pi}{2}x+\pi)-1$, amplitude = 2 , period $= \dfrac{2\pi}{\frac{\pi}{2}} = 4$,

phase shift $= \frac{\pi}{2}x+\pi = 0 \rightarrow x = -2$, vertical displacement = -1



e) If it has a maximum point (2 , 3) and a minimum point of $(6,-7)$, then it has one half period of 4, or a period of 8 ; a vertical displacement of $\frac{-7+3}{2} = -2$; and an amplitude of $\frac{3+7}{2} = 5$

So, $p = \dfrac{2\pi}{b} \rightarrow 8 = \dfrac{2\pi}{b} \rightarrow b = \dfrac{\pi}{4}$. Therefore, $y = 5\sin\dfrac{\pi}{4}x - 2$

f) Since the period of the sine and cosine equation are both 8 with amplitude and vertical displacement the same, the cosine equation is just a shift of two units to the right.

Therefore, the equation is $y = 5\cos\dfrac{\pi}{4}(x-2)-2$ (*Other possible answers*)

g) There are an infinite number of ways to write the answer to this problem. The most common answer is given. This graph has a period of 12; an amplitude of 2; a vertical displacement of 1. The only thing that will change in this problem when writing the equation in terms of sine or cosine, or with $a > 0$ or $a < 0$, is the phase shift.

(*i*) If $a > 0$ then the graph must start at $4 \pm 12n$, n being an integer; let's use 4.
Period $= \frac{2\pi}{|b|} \rightarrow 12 = \frac{2\pi}{b} \rightarrow b = \frac{\pi}{6}$, vertical displacement = 1 , amplitude = 2 .

Therefore, $y = 2\sin\frac{\pi}{6}(x-4)+1$

(*ii*) If $a < 0$ then the graph must start at $-2 \pm 12n$; use -2, therefore, $y = -2\sin\frac{\pi}{6}(x+2)+1$

h) (*i*) If $a > 0$ then the graph must start at $7 \pm 12n$; use 7, $y = 2\cos\frac{\pi}{6}(x-7)+1$

(*ii*) If $a < 0$ then the graph must start at $1 \pm 12n$; use 1, therefore, $y = -2\cos\frac{\pi}{6}(x-1)+1$

Note: The four graphic equations in **16g** and **16h**, above, represent the same graph.

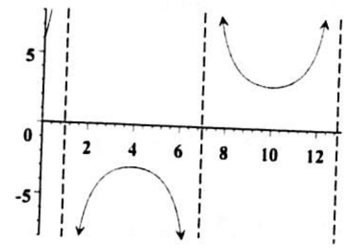
16. i) Period $= \dfrac{2\pi}{\frac{\pi}{6}} = 12$

Phase shift: 1 unit to right

Domain: $x \neq 1 + 6n$, n is an integer

Range: $y \geq 3$ or $y \leq -3$

Asymptote: $x = 1 + 6n$, n is an integer



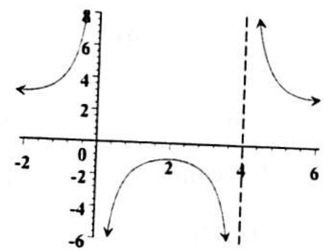
j) Period $= \dfrac{2\pi}{\frac{\pi}{4}} = 8$

Phase shift: 2 units to left

Domain: $x \neq 4n$, n is an integer

Range: $y \geq 3$ or $y \leq -1$

Asymptote: $x = 4n$, n is an integer



17. a) $P = \dfrac{2\pi}{b} \rightarrow \dfrac{1}{60} = \dfrac{2\pi}{b} \rightarrow b = 120\pi$

$E = 110\cos(120\pi t)$

b) *(i)* Period $= \dfrac{60}{20} = 3$ seconds

(ii) $P = \dfrac{2\pi}{b} \rightarrow 3 = \dfrac{2\pi}{b} \rightarrow b = \dfrac{2\pi}{3}$

Vertical displacement $= \dfrac{30 + 8}{2} = 19$ cm, Amplitude $= \dfrac{30 - 8}{2} = 11$ cm

$y = a\cos(bt) + c \rightarrow y = 11\cos(\frac{2\pi}{3}t) + 19$

c) *(i)* Vertical displacement $= 25 + 1 = 26$ m
Maximum height $= 51$ m
Minimum height $= 1$ m
Amplitude $=$ radius $= 25$ m

Period $= \dfrac{2\pi}{b} \rightarrow 24 = \dfrac{2\pi}{b} \rightarrow b = \dfrac{\pi}{12}$, phase shift = 0

$H = 25\sin\left(\dfrac{\pi}{12}t\right) + 26$

(ii) $H = 25\sin\left(\dfrac{\pi}{12} \cdot 16\right) + 26 \rightarrow H = 4.35$ m

(iii) Graph $y = 25\sin\left(\dfrac{\pi}{12}x\right) + 26$ and $y = 35$. Find the intersect of these two equations.
They are $(1.40668, 35)$ and $(10.59332, 35)$ Therefore, the time above 35 m is $10.5932 - 1.40668 = 9.1866$ seconds on each rotation.

17. d) (*i*) Amplitude: $\frac{11.8-4.2}{2} = 3.8$ m , Vertical displacement: $\frac{11.8+4.2}{2} = 8$

Phase shift: 4.5 hours

Period: $(11.5 - 4.5) \cdot 2 = 14$ hours , $P = \frac{2\pi}{b} \rightarrow 14 = \frac{2\pi}{b} \rightarrow b = \frac{\pi}{7}$

$H = -3.8\cos\left(\frac{\pi}{7}(t-4.5)\right) + 8$ or $H = 3.8\sin\left(\frac{\pi}{7}(t-8)\right) + 8$

(*ii*) 1:15 p.m. = 13.25 hours

$H = -3.8\cos\left(\frac{\pi}{7}(13.25-4.5)\right) + 8 \rightarrow H = 10.69$ m or

$H = 3.8\sin\left(\frac{\pi}{7}(13.25-8)\right) + 8 \rightarrow H = 10.69$ m

e) (*i*) Amplitude: 1.1 m Vertical displacement: 1.6 m

Period: 1.0 seconds $P = \frac{2\pi}{b} \rightarrow 1.0 = \frac{2\pi}{b} \rightarrow b = 2\pi$

$H = -1.1\cos(2\pi t) + 1.6$

(*ii*) $H = -1.1\cos(2\pi \cdot 2.3) + 1.6 \rightarrow H = 1.94$ m

f) (*i*) Vertical displacement: 6 m
Maximum: $6 + 8 = 14$ m
Minimum: $6 - 8 = -2$, therefore, zero height of water

(*ii*) Amplitude: 8 m , Vertical displacement: 6 m

Period: 16 minutes $\rightarrow P = \frac{2\pi}{b} \rightarrow 16 = \frac{2\pi}{b} \rightarrow b = \frac{\pi}{8}$

$S = -8\sin\frac{\pi}{8}t + 6$

(*iii*) Since the period is so long, a person in a boat would hardly notice the tsunami.

18. a) $\cos 85° \cos 65° - \sin 85° \sin 65° = \cos(85° + 65°) = \cos 150° = \frac{-\sqrt{3}}{2}$

b) $\sin 82° \cos 22° - \cos 82° \sin 22° = \sin(82° - 22°) = \sin 60° = \frac{\sqrt{3}}{2}$

c) $\sin 75° = \sin(30° + 45°) = \sin 30° \cos 45° + \cos 30° \sin 45°$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$

19. a) $\frac{\sin 3x}{\csc x} - \frac{\cos 3x}{\sec x} = \sin 3x \cdot \sin x - \cos 3x \cdot \cos x = -(\cos 3x \cdot \cos x - \sin 3x \cdot \sin x) = -\cos(3x + x) = -\cos 4x$

b) $\cos(A+B) \cdot \cos B + \sin(A+B) \cdot \sin B \rightarrow \cos(A+B-B) \rightarrow \cos A$

c) $\tan^2(\frac{\pi}{2} - x) \cdot \sec^2 x - \sin^2(\frac{\pi}{2} - x) \cdot \csc^2 x = \cot^2 x \sec^2 x - \cos^2 x \csc^2 x =$

$\frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos^2 x} - \cos^2 x \cdot \frac{1}{\sin^2 x} = \frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x} = \frac{1-\cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\sin^2 x} = 1$

19. d) Remember: $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

Therefore, $\sin(\frac{\pi}{3}-x)\cdot\cos(\frac{\pi}{3}+x) + \cos(\frac{\pi}{3}-x)\cdot\sin(\frac{\pi}{3}+x) = \sin[(\frac{\pi}{3}-x)+(\frac{\pi}{3}+x)] = \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

20. a) $\cos\theta\cos 10° - \sin\theta\sin 10° = \frac{1}{2} \to \cos(\theta+10°) = \frac{1}{2}$, therefore, $\theta + 10° = 60° \to \theta = 50°$

b) $\sin\theta\cdot\cos 12° + \cos\theta\cdot\cos 78° = \frac{\sqrt{3}}{2} \to \sin\theta\cdot\cos 12° + \cos\theta\cdot\sin 12° = \frac{\sqrt{3}}{2}$

$\sin(\theta+12°) = \frac{\sqrt{3}}{2}$, $\sin(60°) = \frac{\sqrt{3}}{2}$, therefore, $\theta+12° = 60° \to \theta = 48°$

21. Remember, $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$y = \cos 3x\cos x - \sin 3x\sin x \to y = \cos(3x+x) \to y = \cos 4x$, therefore, period $= \frac{2\pi}{4} = \frac{\pi}{2}$

22. Remember, $\sin x = \cos(90° - x)$ and $\cos x = \sin(90° - x)$

$\sin 62°\sin 65° + \sin 28°\sin 25° \to \sin 62°\cos 25° + \cos 62°\sin 25° \to \sin(62° + 25°) = \sin 87°$ or $\cos 3°$

23. a) If $\sin A = -\frac{3}{5}$ and in quadrant III, then $\cos A = -\frac{4}{5}$ by Pythagorean theorem.

If $\cos B = -\frac{12}{13}$ and in quadrant III, then $\sin B = -\frac{5}{13}$ by Pythagorean theorem.

$\sin(A-B) = \sin A\cos B - \cos A\sin B = (-\frac{3}{5})(-\frac{12}{13}) - (-\frac{4}{5})(-\frac{5}{13}) = \frac{36}{65} - \frac{20}{65} = \frac{16}{65}$

b) If $\sin A = \frac{12}{13}$ in quadrant II, then by Pythagorean theorem the third side is -5

So $\cos A = -\frac{5}{13}$

If $\sec B = \frac{5}{4}$ in quadrant IV, then by Pythagorean theorem the third side is -3

So $\cos B = \frac{4}{5}$ and $\sin B = -\frac{3}{5}$

$\cos(A+B) = \cos A\cdot\cos B - \sin A\cdot\sin B = (\frac{-5}{13})(\frac{4}{5}) - (\frac{12}{13})(\frac{-3}{5}) = \frac{-20}{65} + \frac{36}{65} = \frac{16}{65}$

c) If $\tan A = \frac{5}{12}$ in quadrant III, then by Pythagorean theorem the hypotenuse is 13

and $\sin A = -\frac{5}{13}$, $\cos A = -\frac{12}{13}$

If $\cos B = -\frac{3}{5}$ in quadrant II then by Pythagorean theorem the third side is 4 and $\sin B = \frac{4}{5}$

So $\sin(A-B) = \sin A\cos B - \cos A\sin B = (-\frac{5}{13})(-\frac{3}{5}) - (-\frac{12}{13})(\frac{4}{5}) = \frac{15}{65} + \frac{48}{65} = \frac{63}{65}$

24. a) $2\sin 67\frac{1}{2}° \cdot \cos 67\frac{1}{2}° = \sin 2(67\frac{1}{2}°) = \sin 135° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

b) $\cos^2 15° - \sin^2 15° = \cos 2(15°) = \cos 30° = \frac{\sqrt{3}}{2}$

c) $4 - 8\cos^2 15° = -4(2\cos^2 15° - 1) = -4\cos 2(15°) = -4\cos 30° = -4\times\frac{\sqrt{3}}{2} = -2\sqrt{3}$

d) $3\sin^2\frac{\pi}{8} - 3\cos^2\frac{\pi}{8} = -3\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right) = -3\cos 2\left(\frac{\pi}{8}\right) = -3\cos\frac{\pi}{4} = \frac{-3\sqrt{2}}{2}$

e) $2 - 4\sin^2\frac{7\pi}{12} = 2\left(1 - 2\sin^2\frac{7\pi}{12}\right) = 2\cos 2\left(\frac{7\pi}{12}\right) = 2\cos\frac{7\pi}{6} = 2\left(\frac{-\sqrt{3}}{2}\right) = -\sqrt{3}$

f) $2\cos^2 15° + 1 = (2\cos^2 15° - 1) + 2 = \cos 2(15°) + 2 = \cos 30° + 2 = \frac{\sqrt{3}}{2} + 2$

25. a) If $\tan\theta = -\frac{2}{3}$ in quadrant II, therefore, $r = \sqrt{13}$ by Pythagoras Theorem

$$\csc 2\theta = \frac{1}{\sin 2\theta} = \frac{1}{2\sin\theta\cos\theta} = \frac{1}{2\left(\frac{2}{\sqrt{13}}\right)\left(\frac{-3}{\sqrt{13}}\right)} = \frac{1}{\frac{-12}{13}} = \frac{-13}{12}$$

b) If $\sin\theta = \frac{3}{5}$ then $\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\left(\frac{3}{5}\right)^2 = \frac{7}{25}$

c) If $\cos\theta = \frac{12}{13}$ then, by Pythagoras Theorem, $\sin\theta = \frac{-5}{13}$ in quadrant III

So $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right) = \frac{120}{169}$

d) If $\tan\theta = -\frac{2}{3}$ then, by Pythagoras Theorem, $a^2 + b^2 = c^2 \rightarrow (2)^2 + (-3)^2 = c^2 \rightarrow c = \sqrt{13}$

Then $\sin\theta = \frac{-2}{\sqrt{13}}$ and $\cos\theta = \frac{3}{\sqrt{13}}$

So $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{-2}{\sqrt{13}}\right)\left(\frac{3}{\sqrt{13}}\right) = \frac{-12}{13}$

e) If $\tan\theta = \frac{x}{1}$ then the hypotenuse of this triangle is $\sqrt{x^2+1}$

Therefore, $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{x}{\sqrt{x^2+1}}\right)\left(\frac{1}{\sqrt{x^2+1}}\right) = \frac{2x}{x^2+1}$

f) If $\sin\theta = \frac{x}{3}$, $\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\left(\frac{x}{3}\right)^2 = 1 - \frac{2x^2}{9}$

26. a) Remember $\sin 2x = 2\sin x\cos x$, so $y = 4\sin 3x\cos 3x = 2\sin 6x$

Therefore, amplitude $= 2$, period $= \frac{2\pi}{6} = \frac{\pi}{3}$

b) $f(x) = 4\cos^2 3x - 2 \rightarrow f(x) = 2(2\cos^2 3x - 1) = 2\cos 6x$

Therefore, amplitude $= |2| = 2$, period $= \frac{2\pi}{|6|} = \frac{\pi}{3}$

27. Remember, every time you go from 0° to 360°, or from 0 to 2π, each trigonometric function is positive in two quadrants and negative in two quadrants. But if the trigonometric value is found on the x-axis or y-axis, it can have one or two solutions.

<u>*Example*</u>: $\cos\theta = -1$ only at 180°, but $\cos\theta = 0$ at both 90° and 270°.

a) $\sin 3x = -\frac{1}{4}$. Sine has a value of $-\frac{1}{4}$ in quadrants III, IV. Since the period is $\frac{2\pi}{3}$, to go to 2π requires multiplying $\frac{2\pi}{3}$ by 3, therefore, $2 \times 3 = 6$ solutions.

b) $\sin 3x = -1$. Sine has a value of -1 only at $\frac{3\pi}{2}$. Since the period is $\frac{2\pi}{3}$, to go to 2π requires multiplying $\frac{2\pi}{3}$ by 3, therefore, $1 \times 3 = 3$ solutions.

c) $\sin\frac{1}{2}x = \frac{1}{3}$. Sine has a value of $\frac{1}{3}$ in quadrants I, II. Since the period is $\frac{2\pi}{\frac{1}{2}} = 4\pi$, to go to 2π requires multiplying 4π by $\frac{1}{2}$, therefore, $\frac{1}{2}$ the period fines solutions in quadrants I, II, so 2 solutions

d) $\cos\frac{1}{2}x = \frac{1}{3}$. Cosine has a value of $\frac{1}{3}$ in quadrants I, IV. Since the period is $\frac{2\pi}{\frac{1}{2}} = 4\pi$, to go to 2π requires multiplying 4π by $\frac{1}{2}$, therefore, $\frac{1}{2}$ the period fines a solution in quadrants I, so 1 solution.

e) $\tan^2 2x = 1 \rightarrow \tan 2x = \pm 1$. Tangent has a positive or negative value in all 4 quadrants. The period of $\frac{\pi}{2}$ gives a positive and negative value of tangent, $\frac{\pi}{2} \times 4 = 2\pi$ would give $2 \times 4 = 8$ solutions.

f) $\sin bx = \frac{1}{2}$. Sine has a value of $\frac{1}{2}$ in quadrants I, II. Since the period is $\frac{2\pi}{b}$, to go to 2π requires multiplying $\frac{2\pi}{b}$ by b, therefore, $2 \times b = 2b$ solutions

28. a)

$$\frac{\sec x}{1-\cos x} = \frac{\sec x+1}{\sin^2 x}$$

$$\frac{\sec x}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x} =$$

$$\frac{\sec x+1}{1-\cos^2 x} =$$

$$\frac{\sec x+1}{\sin^2 x} =$$

b)

$$\frac{\sin\theta+\cos\theta\cdot\cot\theta}{\cos\theta\csc\theta} = \sec\theta$$

$$\frac{\sin\theta+\cos\theta\cdot\dfrac{\cos\theta}{\sin\theta}}{\dfrac{\cos\theta}{\sin\theta}} = \frac{1}{\cos\theta}$$

$$\frac{\sin^2\theta+\cos^2\theta}{\sin\theta}\cdot\frac{\sin\theta}{\cos\theta} =$$

$$\frac{1}{\cos\theta} =$$

c)

$$\frac{1+\sec\theta}{\sin\theta+\tan\theta} = \csc\theta$$

$$\frac{1+\dfrac{1}{\cos\theta}}{\sin\theta+\dfrac{\sin\theta}{\cos\theta}} =$$

$$\frac{\dfrac{\cos\theta+1}{\cos\theta}}{\dfrac{\sin\theta\cdot\cos\theta+\sin\theta}{\cos\theta}} =$$

$$\frac{\cos\theta+1}{\sin\theta(\cos\theta+1)} =$$

$$\csc\theta =$$

d)

$$\frac{\sec x}{1-\sin x} = \frac{1+\sin x}{\cos^3 x}$$

$$\frac{\sec x(1+\sin x)}{(1-\sin x)(1+\sin x)} =$$

$$\frac{\sec x(1+\sin x)}{1-\sin^2 x} =$$

$$\frac{\sec x(1+\sin x)}{\cos^2 x} =$$

$$\frac{1+\sin x}{\cos^3 x} =$$

e)

$$\cos^2 x = \frac{1-2\sin^2 x}{1-\tan^2 x}$$

$$= \frac{1-2\sin^2 x}{1-\dfrac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{1-2\sin^2 x}{\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}}$$

$$= \frac{\cos^2 x(1-2\sin^2 x)}{\cos^2 x-\sin^2 x}$$

$$= \frac{\cos^2 x(1-2\sin^2 x)}{1-\sin^2 x-\sin^2 x}$$

$$= \frac{\cos^2 x(1-2\sin^2 x)}{1-2\sin^2 x}$$

$$= \cos^2 x$$

f)

$$\tan x = \csc 2x - \cot 2x$$

$$= \frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x}$$

$$= \frac{1-\cos 2x}{\sin 2x}$$

$$= \frac{1-(1-2\sin^2 x)}{2\sin x\cos x}$$

$$= \frac{2\sin^2 x}{2\sin x\cos x}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

g)

$$\frac{1-\cos\theta}{\sin\theta} = \frac{1}{\csc\theta+\cot\theta}$$

$$\frac{1-\cos\theta}{\sin\theta}\cdot\frac{(1+\cos\theta)}{(1+\cos\theta)} = \frac{1}{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}}$$

$$\frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)} = \frac{1}{\frac{1+\cos\theta}{\sin\theta}}$$

$$\frac{\sin^2\theta}{\sin\theta(1+\cos\theta)} = \frac{\sin\theta}{1+\cos\theta}$$

$$\frac{\sin\theta}{1+\cos\theta} =$$

h)

$$\frac{\cos 2\theta}{\sin\theta}+\frac{\sin 2\theta}{\cos\theta} = \csc\theta$$

$$\frac{\cos 2\theta\cdot\cos\theta+\sin 2\theta\cdot\sin\theta}{\sin\theta\cos\theta} =$$

$$\frac{\cos(2\theta-\theta)}{\sin\theta\cos\theta} =$$

$$\frac{\cos\theta}{\sin\theta\cos\theta} =$$

$$\frac{1}{\sin\theta} =$$

$$\csc\theta =$$

i)

$$\frac{\sin^2 x-\tan x}{\cos^2 x-\cot x} = \tan^2 x$$

$$\frac{\sin^2 x-\frac{\sin x}{\cos x}}{\cos^2 x-\frac{\cos x}{\sin x}} = \frac{\sin^2 x}{\cos^2 x}$$

$$\frac{\frac{\sin^2 x\cos x-\sin x}{\cos x}}{\frac{\sin x\cos^2 x-\cos x}{\sin x}} =$$

$$\frac{\sin x(\sin x\cos x-1)}{\cos x(\sin x\cos x-1)}\cdot\frac{\sin x}{\cos x} =$$

$$\frac{\sin^2 x}{\cos^2 x} =$$

j)

$$\cos 2x = \frac{\cot x-\tan x}{\cot x+\tan x}$$

$$= \frac{\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}}$$

$$= \frac{\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}}{\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}}$$

$$= \frac{\cos^2 x-\sin^2 x}{\sin^2 x+\cos^2 x}$$

$$= \cos 2x$$

k)

$$\csc x-\frac{\sin x}{1+\cos x} = \cot x$$

$$\frac{1}{\sin x}-\frac{\sin x}{1+\cos x} = \frac{\cos x}{\sin x}$$

$$\frac{1+\cos x-\sin^2 x}{\sin x(1+\cos x)} =$$

$$\frac{\cos x+\cos^2 x}{\sin x(1+\cos x)} =$$

$$\frac{\cos x(1+\cos x)}{\sin x(1+\cos x)} =$$

$$\frac{\cos x}{\sin x} =$$

l)

$$\cot x-\tan x = \frac{4\cos^2 x-2}{\sin 2x}$$

$$\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x} = \frac{2(2\cos^2 x-1)}{\sin 2x}$$

$$\frac{\cos^2 x-\sin^2 x}{\sin x\cos x} = \frac{2\cos 2x}{\sin 2x}$$

$$\frac{\cos 2x}{\frac{1}{2}\sin 2x} =$$

$$\frac{2\cos 2x}{\sin 2x} =$$

28. **m)**

$$\frac{1-\sin x}{1+\sin x}=(\sec x-\tan x)^2$$

$$=\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)^2$$

$$=\frac{(1-\sin x)^2}{\cos^2 x}$$

$$=\frac{(1-\sin x)^2}{1-\sin^2 x}$$

$$=\frac{(1-\sin x)^2}{(1-\sin x)(1+\sin x)}$$

$$=\frac{1-\sin x}{1+\sin x}$$

n)

$$\frac{\sin 4x-\sin 2x}{\cos 4x+\cos 2x}=\tan x$$

$$\frac{2\sin 2x\cos 2x-\sin 2x}{2\cos^2 2x-1+\cos 2x}=\frac{\sin x}{\cos x}$$

$$\frac{\sin 2x(2\cos 2x-1)}{(2\cos 2x-1)(\cos 2x+1)}=$$

$$\frac{2\sin x\cos x}{(2\cos^2 x-1)+1}=$$

$$\frac{2\sin x\cos x}{2\cos^2 x}=$$

$$\frac{\sin x}{\cos x}=$$

o)

$$\tan 2x=\frac{2}{\cot x-\tan x}$$

$$\frac{\sin 2x}{\cos 2x}=\frac{2}{\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}}$$

$$=\frac{2}{\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}}$$

$$=\frac{2\sin x\cos x}{\cos^2 x-\sin^2 x}$$

$$=\frac{\sin 2x}{\cos 2x}$$

p)

$$\frac{\cot x-\cos x}{1-\sin x}=\frac{\sin 2x}{1-\cos 2x}$$

$$\frac{\frac{\cos x}{\sin x}-\cos x}{1-\sin x}=\frac{2\sin x\cos x}{1-\left(1-2\sin^2 x\right)}$$

$$\frac{\cos x-\sin x\cos x}{\sin x(1-\sin x)}=\frac{2\sin x\cos x}{2\sin^2 x}$$

$$\frac{\cos x(1-\sin x)}{\sin x(1-\sin x)}=\frac{\cos x}{\sin x}$$

$$\cot x=\cot x$$

q)

$$\frac{\cos x}{\csc x+1}+\frac{\cos x}{\csc x-1}=2\tan x$$

$$\frac{\cos x(\csc x-1)+\cos x(\csc x+1)}{\csc^2 x-1}=\frac{2\sin x}{\cos x}$$

$$\frac{\cos x\csc x-\cos x+\cos x\csc x+\cos x}{\csc^2 x-1}=$$

$$\frac{2\cos x\csc x}{\cot^2 x}=$$

$$\frac{2\cos x}{\sin x}\cdot\frac{\sin^2 x}{\cos^2 x}=$$

$$\frac{2\sin x}{\cos x}=$$

r)

$$\frac{\cos 2x}{1-\sin 2x}=\frac{1+\tan x}{1-\tan x}$$

$$=\frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}$$

$$=\frac{\frac{\cos x+\sin x}{\cos x}}{\frac{\cos x-\sin x}{\cos x}}$$

$$=\frac{(\cos x+\sin x)(\cos x-\sin x)}{(\cos x-\sin x)(\cos x-\sin x)}$$

$$=\frac{\cos^2 x-\sin^2 x}{\cos^2 x-2\sin x\cos x+\sin^2 x}$$

$$=\frac{\cos 2x}{1-\sin 2x}$$

28. s) $\tan x(\csc x+1)=\dfrac{\cot x}{\csc x-1}$

$$\frac{\csc x+1}{\cot x}=\frac{\cot x}{(\csc x-1)}\cdot\frac{(\csc x+1)}{(\csc x+1)}$$
$$=\frac{\cot x(\csc x+1)}{\csc^2 x-1}$$
$$=\frac{\cot x(\csc x+1)}{\cot^2 x}$$
$$=\frac{\csc x+1}{\cot x}$$

t) $\cot 2x=\dfrac{\cot^2 x-1}{2\cot x}$

$$\frac{\cos 2x}{\sin 2x}=\frac{\tan x}{2}\left(\frac{\cos^2 x}{\sin^2 x}-1\right)$$
$$=\frac{\sin x}{2\cos x}\left(\frac{\cos^2 x-\sin^2 x}{\sin^2 x}\right)$$
$$=\frac{\cos^2 x-\sin^2 x}{2\sin x\cos x}$$
$$=\frac{\cos 2x}{\sin 2x}$$

29. a) (*i*) $\sin x>0$ in quadrant I, II, $x=\sin^{-1}(0.6234)=0.673$, $x=\pi-0.673=2.469$.
(*ii*) $x=0.673+2\pi n$, n an integer, $x=2.469+2\pi n$, n an integer

b) (*i*) $\cos x>0$ in quadrant I, IV, $x=\cos^{-1}(0.4821)=1.068$, $x=2\pi-1.068=5.215$.
(*ii*) $x=1.068+2\pi n$, n an integer, $x=5.215+2\pi n$, n an integer

c) (*i*) $\tan x>0$ in quadrant I, III, $x=\tan^{-1}(1.7258)=1.046$, $x=\pi+1.046=4.187$.
(*ii*) Because of tangent symmetry, $x=1.046+\pi n$, n an integer

d) (*i*) $\cot x>0$ in quadrant I, III, $x=\tan^{-1}\left(\frac{1}{0.7238}\right)=0.944$, $x=\pi+0.944=4.086$.
(*ii*) Because of cotangent symmetry, $x=0.944+\pi n$, n an integer

e) (*i*) $\sec x>0$ in quadrant I, IV, $x=\cos^{-1}\left(\frac{1}{3.1743}\right)=1.250$, $x=2\pi-1.250=5.033$.
(*ii*) $x=1.250+2\pi n$, n an integer, $x=5.033+2\pi n$, n an integer

f) (*i*) $\csc x>0$ in quadrant I, II, $x=\sin^{-1}\left(\frac{1}{1.5243}\right)=0.716$, $x=\pi-0.716=2.426$.
(*ii*) $x=0.716+2\pi n$, n an integer, $x=2.426+2\pi n$, n an integer

g) (*i*) $\sin x<0$ in quadrant III, IV, reference angle is $x=\sin^{-1}(0.4173)=0.430$,
therefore, answers are $x=\pi+0.430=3.572$, $x=2\pi-0.430=5.853$.
(*ii*) $x=3.572+2\pi n$, n an integer, $x=5.853+2\pi n$, n an integer

h) (*i*) $\cos x<0$ in quadrant II, III, reference angle is $x=\cos^{-1}(0.4821)=1.068$,
therefore, answers are $x=\pi-1.068=2.074$, $x=\pi+1.068=4.209$.
(*ii*) $x=2.074+2\pi n$, n an integer, $x=4.209+2\pi n$, n an integer

i) (*i*) $\tan x<0$ in quadrant II, IV, reference angle is $x=\tan^{-1}(0.3124)=0.303$,
therefore, answers are $x=\pi-0.303=2.839$, $x=2\pi-0.303=5.980$.
(*ii*) Because of tangent symmetry, $x=2.839+\pi n$, n an integer

j) (*i*) $\cot x<0$ in quadrant II, IV, reference angle is $x=\tan^{-1}\left(\frac{1}{1.1482}\right)=0.717$,
therefore, answers are $x=\pi-0.717=2.425$, $x=2\pi-0.717=5.567$.
(*ii*) Because of cotangent symmetry, $x=2.425+\pi n$, n an integer

k) (*i*) $\sec x<0$ in quadrant II, III, reference angle is $x=\cos^{-1}\left(\frac{1}{1.9105}\right)=1.020$,
therefore, answers are $x=\pi-1.020=2.122$, $x=\pi+1.020=4.162$.
(*ii*) $x=2.122+2\pi n$, n an integer, $x=4.162+2\pi n$, n an integer

l) (*i*) $\csc x<0$ in quadrant III, IV, reference angle is $x=\sin^{-1}\left(\frac{1}{2.3124}\right)=0.447$,
therefore, answers are $x=\pi+0.447=3.589$, $x=2\pi-0.447=5.836$.
(*ii*) $x=3.589+2\pi n$, n an integer, $x=5.836+2\pi n$, n an integer

29) m)(*i*) $\sin 2x = -0.4173$. Sine has a value of -0.4173 in quadrants III, IV. Radian values are 3.572, 5.853
The period is $\frac{2\pi}{2} = \pi$, therefore, 2π requires multiplying π by 2, therefore, $2 \times 2 = 4$ solutions.
Therefore, $2x = 3.572$, $2x = 5.853$, $2x = 3.572 + 2\pi = 9.855$, $2x = 5.853 + 2\pi = 12.136$
$x = 1.786$, $x = 2.926$, $x = 4.928$, $x = 6.068$

(*ii*) The general solution is $2x = 3.572 + 2\pi n$, $2x = 5.853 + 2\pi n$
$x = 1.786 + \pi n$, n an integer $x = 2.926 + \pi n$, n an integer

n) (*i*) $\tan 2x = 1.7258$. Tangent has a value of 1.7258 in quadrants I, III. Radian values are 1.046, 4.187
The period is $\frac{2\pi}{2} = \pi$, therefore, 2π requires multiplying π by 2, therefore, $2 \times 2 = 4$ solutions.
Therefore, $2x = 1.046$, $2x = 4.187$, $2x = 1.046 + 2\pi = 7.329$, $2x = 4.187 + 2\pi = 10.470$
$x = 0.523$, $x = 2.094$, $x = 3.664$, $x = 5.235$

(*ii*) The general solution is $2x = 1.046 + \pi n$,
$x = 0.523 + \frac{\pi}{2} n$, n an integer

o) (*i*) $\tan bx = 1.7258$ Tangent has a value of 1.7258 in quadrants I, III. Radian values are 1.046, 4.187
Therefore, $bx = 1.046$, $bx = 4.187$, $bx = 1.046 + 2\pi = 7.329$, $bx = 4.187 + 2\pi = 10.470$
$x = \frac{1.046}{b}$, $x = \frac{4.187}{b}$, $x = \frac{7.329}{b}$, $x = \frac{10.470}{b}, \ldots\ldots\ldots < 2\pi$

(*ii*) The general solution is $bx = 1.046 + \pi n$,
$x = \frac{1.046}{b} + \frac{\pi}{b} n$, n an integer

30. a) (*i*) $2\cos x + 1 = 0 \to \cos x = \frac{-1}{2}$, $\cos x < 0$ in quadrant II, III, by special angles at $x = \frac{2\pi}{3}, \frac{4\pi}{3}$.

(*ii*) $x = \frac{2\pi}{3} + 2\pi n$, n an integer, $x = \frac{4\pi}{3} + 2\pi n$, n an integer

b) (*i*) $(2\sin x - 1)(\cos x + 1) = 0 \to \sin x = \frac{1}{2}$ at $\frac{\pi}{6}, \frac{5\pi}{6}$, $\cos x = -1$ at π. Therefore, $x = \frac{\pi}{6}, \frac{5\pi}{6}, \pi$.

(*ii*) $x = \frac{\pi}{6} + 2\pi n$, n an integer, $x = \frac{5\pi}{6} + 2\pi n$, n an integer, $x = \pi + 2\pi n$, n an integer

c) (*i*) $\sqrt{2}\cos^2 x - \cos x = 0 \to \cos x(\sqrt{2}\cos x - 1) = 0 \to \cos x = 0$ at $\frac{\pi}{2}, \frac{3\pi}{2}$,
$\cos x = \frac{1}{\sqrt{2}}$ at $\frac{\pi}{4}, \frac{7\pi}{4}$. Therefore, $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{4}$

(*ii*) $x = \frac{\pi}{2} + \pi n$, n an integer, $x = \frac{\pi}{4} + 2\pi n$, n an integer, $x = \frac{7\pi}{4} + 2\pi n$, n an integer

d) (*i*) $4\sin^2 x = 3 \to \sin^2 x = \frac{3}{4} \to \sin x = \frac{\pm\sqrt{3}}{2}$ is found in all 4 quadrants at $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$.

(*ii*) $x = \frac{\pi}{3} + \pi n$, n an integer, $x = \frac{2\pi}{3} + \pi n$, n an integer

e) (*i*) $\sin^2 x = \sin x \to \sin^2 x - \sin x = 0 \to \sin x(\sin x - 1) = 0 \to \sin x = 0$, $\sin x = 1$
$\sin x = 0$ at $0, \pi$, $\sin x = 1$ at $\frac{\pi}{2}$ Therefore, answers are $x = 0, \frac{\pi}{2}, \pi$

(*ii*) $x = \pi n$, n an integer, $x = \frac{\pi}{2} + 2\pi n$, n an integer

f) (*i*) $6\sin^2 x + 11\sin x - 10 = 0 \to (3\sin x - 2)(2\sin x + 5) = 0 \to \sin x = \frac{2}{3}, -\frac{5}{2}$. $\sin x = -\frac{5}{2} \to x = \phi$
sine is positive in quadrants I, II. So, $x = 0.730$, $x = \pi - 0.730 = 2.412$

(*ii*) $x = 0.730 + 2\pi n$, n an integer, $x = 2.412 + 2\pi n$, n an integer

g) (*i*) $5\cos^2 x + 6\cos x - 8 = 0 \to (5\cos x - 4)(\cos x + 2) = 0 \to \cos x = \frac{4}{5}, -2$, $\cos x = -2 \to x = \phi$
Cosine is positive in quadrants I, IV. $x = \cos^{-1}\left(\frac{4}{5}\right) = 0.644$, $x = 2\pi - 0.644 = 5.640$

(*ii*) $x = 0.644 + 2\pi n$, n an integer, $x = 5.640 + 2\pi n$, n an integer

30. h) (*i*) $2\cos^2 x - \cos x - 1 = 0 \rightarrow (2\cos x + 1)(\cos x - 1) = 0 \rightarrow \cos x = -\frac{1}{2},\ 1$

$\cos x = -\frac{1}{2}$ at $\frac{2\pi}{3},\ \frac{4\pi}{3}$, and $\cos x = 1$ at 0

(*ii*) Because of the symmetry of $0,\ \frac{2\pi}{3},\ \frac{4\pi}{3}$, $x = \frac{2\pi}{3}n$, n an integer

i) (*i*) $2\cos^2 x - 3\cos x - 2 = 0 \rightarrow (2\cos x + 1)(\cos x - 2) = 0 \rightarrow \cos x = -\frac{1}{2},\ 2$

$\cos x = 2 \rightarrow x = \phi$, cosine has a value of $-\frac{1}{2}$ in quadrant II, III. Radian values are $\frac{2\pi}{3},\ \frac{4\pi}{3}$

(*ii*) $x = \frac{2\pi}{3} + 2\pi n$, n an integer, $x = \frac{4\pi}{3} + 2\pi n$, n an integer

j) (*i*) $2\tan^2 x + 5\tan x + 2 = 0 \rightarrow (2\tan x + 1)(\tan x + 2) = 0 \rightarrow \tan x = -\frac{1}{2},\ -2$

$\tan x = -\frac{1}{2} \rightarrow x = \tan^{-1}\left(\frac{1}{2}\right) = 0.464 \rightarrow$ $x = \pi - 0.464 = 2.678$, $x = 2\pi - 0.464 = 5.820$

$\tan x = -2 \rightarrow x = \tan^{-1}(2) = 1.107 \rightarrow$ $x = \pi - 1.107 = 2.034$, $x = 2\pi - 1.107 = 5.176$

(*ii*) $x = 2.678 + \pi n$, n an integer, $x = 2.034 + \pi n$, n an integer

k) (*i*) $\tan^2 x - 2\tan x - 3 = 0 \rightarrow (\tan x + 1)(\tan x - 3) = 0 \rightarrow \tan x = -1,\ 3$

$\tan x = -1$ in quadrants II, IV. The special angle values are $\frac{3\pi}{4},\ \frac{7\pi}{4}$

$\tan x = 3$ in quadrants I, III. $x = \tan^{-1}(3) = 1.249$, $x = \pi + 1.249 = 4.391$

(*ii*) $x = \frac{3\pi}{4} + \pi n$, n an integer, $x = 1.249 + \pi n$, n an integer

l) (*i*) $\cot^2 x - \cot x - 6 = 0 \rightarrow (\cot x - 3)(\cot x + 2) = 0 \rightarrow \cot x = 3,\ -2$. $\cot x = -2$ in quadrants II, IV.

$x = \cot^{-1}(2) = \tan^{-1}\left(\frac{1}{2}\right) = 0.464$, $x = \pi - 0.464 = 2.678$, $x = 2\pi - 0.464 = 5.820$

$\cot x = 3$ in quadrants I, III. $x = \cot^{-1}(3) = \tan^{-1}\left(\frac{1}{3}\right) = 0.322$, $x = \pi + 0.322 = 3.463$

(*ii*) $x = 2.678 + \pi n$, n an integer, $x = 0.322 + \pi n$, n an integer

m) (*i*) $\tan x - 2\tan x \cdot \sin x = 0 \rightarrow \tan x(1 - 2\sin x) = 0$, $\tan x = 0$ at $0,\ \pi$, $\sin x = \frac{1}{2}$ at $\frac{\pi}{6},\ \frac{5\pi}{6}$.

(*ii*) $x = \pi n$, n an integer, $x = \frac{\pi}{6} + 2\pi n$, n an integer, $x = \frac{5\pi}{6} + 2\pi n$, n an integer

n) (*i*) $3\sin^2 x + 4\sin x - 4 = 0 \rightarrow (3\sin x - 2)(\sin x + 2) = 0 \rightarrow \sin x = \frac{2}{3},\ -2$. $\sin x = -2 \rightarrow x = \phi$

$\sin x = \frac{2}{3}$ in quadrants I, II. $x = \sin^{-1}\left(\frac{2}{3}\right) = 0.730$, $x = \pi - 0.730 = 2.412$

(*ii*) $x = 0.730 + 2\pi n$, n an integer, $x = 2.412 + 2\pi n$, n an integer

o) (*i*) $\sec^2 x - 3\sec x + 2 = 0 \rightarrow (\sec x - 1)(\sec x - 2) = 0 \rightarrow \sec x = 1$ at 0, $\sec x = 2$ at $\frac{\pi}{3},\ \frac{5\pi}{3}$.

(*ii*) $x = 2\pi n$, n an integer, $x = \frac{\pi}{3} + 2\pi n$, n an integer, $x = \frac{5\pi}{3} + 2\pi n$, n an integer

p) (*i*) $2\cos^2 x - 3\sin x - 3 = 0 \rightarrow 2(1 - \sin^2 x) - 3\sin x - 3 = 0 \rightarrow 2 - 2\sin^2 x - 3\sin x - 3 = 0 \rightarrow$

$2\sin^2 x + 3\sin x + 1 = 0 \rightarrow (2\sin x + 1)(\sin x + 1) = 0 \rightarrow \sin x = -\frac{1}{2},\ -1$

$\sin x = -\frac{1}{2}$ at $\frac{7\pi}{6},\ \frac{11\pi}{6}$, $\sin x = -1$ at $\sin x = -1$ *at* $\frac{3\pi}{2}$

(*ii*) $x = \frac{7\pi}{6} + 2\pi n$, n an integer, $x = \frac{11\pi}{6} + 2\pi n$, n an integer, $x = \frac{3\pi}{2} + 2\pi n$, n an integer

q) (*i*) $3\csc x - \sin x - 2 = 0 \rightarrow \left(\frac{3}{\sin x} - \sin x - 2 = 0\right)\sin x \rightarrow 3 - \sin^2 x - 2\sin x = 0 \rightarrow \sin^2 x + 2\sin x - 3 = 0$

$(\sin x - 1)(\sin x + 3) = 0 \rightarrow \sin x = 1,\ -3$. $\sin x = -3 \rightarrow x = \phi$, $\sin x = 1$ *at* $\frac{\pi}{2}$

(*ii*) $x = \frac{\pi}{2} + 2\pi n$, n an integer

r) (*i*) $\sin 2x - 2\cos x = 0 \rightarrow 2\sin x\cos x - 2\cos x = 0 \rightarrow 2\cos x(\sin x - 1) = 0 \rightarrow \cos x = 0,\ \sin x = 1$

$\cos x = 0$ *at* $\frac{\pi}{2},\ \frac{3\pi}{2}$, $\sin x = 1$ *at* $\frac{\pi}{2}$, therefore answers are $\frac{\pi}{2},\ \frac{3\pi}{2}$

(*ii*) $x = \frac{\pi}{2} + \pi n$, n an integer

30. **s)** (*i*) $\sin x \tan 2x = \sin x \rightarrow \sin x \tan 2x - \sin x = 0 \rightarrow \sin x(\tan 2x - 1) = 0 \rightarrow \sin x = 0,\ \tan 2x = 1$

Tangent equals 1 at $\frac{\pi}{4}$, $\frac{5\pi}{4} \rightarrow 2x = \frac{\pi}{4}$, $2x = \frac{5\pi}{4}$, $2x = \frac{\pi}{4} + 2\pi = \frac{9\pi}{4}$, $2x = \frac{5\pi}{4} + 2\pi = \frac{13\pi}{4}$

$x = \frac{\pi}{8}$ $\quad x = \frac{5\pi}{8}$ $\quad x = \frac{9\pi}{8}$ $\quad x = \frac{13\pi}{8}$

$\sin x = 0$ at $0, \pi$. Therefore answers are $0, \pi, \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

(*ii*) $2x = \frac{\pi}{4} + \pi n \rightarrow x = \frac{\pi}{8} + \frac{\pi}{2}n$, n an integer, $\quad \pi n$, n an integer

t) (*i*) $3\sin^2 2x - 2\sin 2x - 1 = 0 \rightarrow (\sin 2x - 1)(3\sin 2x + 1) = 0 \rightarrow \sin 2x = -\frac{1}{3},\ 1$

Sine has a value of $-\frac{1}{3}$ in quadrants III, IV. Radian values are 3.481, 5.943

Therefore $2x = 3.481$, $\quad 2x = 5.954$, $\quad 2x = 3.481 + 2\pi$, $\quad 2x = 5.943 + 2\pi$

$x = 1.741$ $\quad x = 2.977$ $\quad x = 4.883$ $\quad x = 6.113$

Sine has a value of 1 at $\frac{\pi}{2}$, therefore $2x = \frac{\pi}{2} \rightarrow x = \frac{\pi}{4}$, $\quad 2x = \frac{\pi}{2} + 2\pi = \frac{5\pi}{2} \rightarrow x = \frac{5\pi}{4}$

Therefore, answers are 1.741, 2.977, 4.883, 6.113, $\frac{\pi}{4}$, and $\frac{5\pi}{4}$

(*ii*) $2x = 3.481 + 2\pi n$ $\qquad 2x = 5.943 + 2\pi n$ $\qquad 2x = \frac{\pi}{2} + 2\pi n$

$x = 1.741 + \pi n$, n an integer, $\quad x = 2.977 + \pi n$, n an integer, $\quad x = \frac{\pi}{4} + \pi n$, n an integer

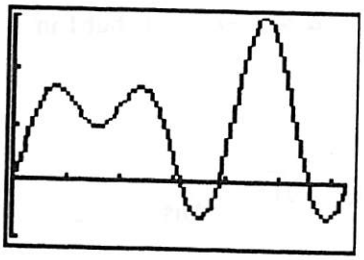
31. **a)**

$x\ [0, 2\pi]$ $\qquad y\ [-8, 8]$

$Y_1 = \tan x - \sin 3x - 1$

The solution is the crossing of the x-axis.
Therefore, the solutions are $x = 0.9306, 3.4113$

b)



$x\ [0, 2\pi]$ $\qquad y\ [-1, 3]$

$Y_1 = \sin 3x - \cos 2x + 1$

The solution is the crossing of the x-axis.
Therefore, the solutions are $x = 0, 3.1416,$
$3.8510, 5.5738$

c)

$x\ [0, 2\pi]$ $\qquad y\ [-4, 4]$

$Y_1 = \dfrac{1}{\tan 2x} + \tan 0.5x$

The solution is the crossing of the x-axis.
Therefore, the solutions are $x = 1.0472, 5.2360$

Trigonometry – Multiple-choice Solutions

ANSWERS

1. d	9. c	17. c	25. b	33. c	41. d	49. c	57. c	65. b	73. c
2. d	10. a	18. a	26. c	34. d	42. a	50. c	58. a	66. a	74. c
3. c	11. a	19. c	27. a	35. d	43. a	51. b	59. c	67. b	75. a
4. d	12. b	20. d	28. c	36. b	44. d	52. a	60. a	68. b	
5. c	13. a	21. b	29. b	37. c	45. b	53. a	61. b	69. a	
6. d	14. a	22. b	30. d	38. a	46. d	54. d	62. b	70. a	
7. d	15. d	23. a	31. c	39. a	47. b	55. d	63. c	71. c	
8. c	16. b	24. c	32. d	40. c	48. a	56. b	64. d	72. c	

SOLUTIONS

1. Amplitude $|-2| = 2$, period $\frac{2\pi}{\frac{\pi}{2}} = 4$. Answer is d.

2. Phase shift $\frac{\pi}{2}x - \pi = 0 \rightarrow \frac{\pi}{2}x = \pi \rightarrow x = 2$, vertical displacement 3. Answer is d.

3. $Period = \frac{\pi}{b} = \frac{\pi}{\frac{\pi}{3}} = 3$. Answer is c.

4. $a = r\theta$, $a =$ arc length, $r =$ radius, $\theta =$ angle in radians

   $12 = 6\theta \rightarrow \theta = 2$ radians, degree angle $2 \cdot \frac{180^\circ}{\pi} = 115^\circ$. Answer is d.

5. Cosecant is negative in quadrant III and IV, tangent is negative in quadrant II and IV therefore, solution is in quadrant IV.

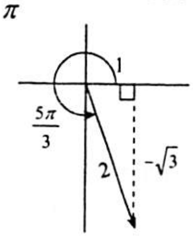
   $\csc x = \frac{hyp}{opp} = \frac{2}{-\sqrt{3}}$, by Pythagoras Theorem $\left(-\sqrt{3}\right)^2 + a^2 = 2^2 \rightarrow a^2 = 1 \rightarrow a \pm 1$, but 1 in Quadrant IV, therefore, $\cos x = \frac{adj}{hyp} = \frac{1}{2}$. Answer is c.

6. By Pythagoras Theorem $r^2 = 2^2 + (-3)^2 \rightarrow r = \sqrt{13}$, $\sec\theta = \frac{hyp}{adj} = \frac{\sqrt{13}}{2}$. Answer is d.

7. $1 - \cos 2x = 1 - (1 - 2\sin^2 x) = 1 - 1 + 2\sin^2 x = 2\sin^2 x$. Answer is d.

8. The smallest negative angle for cosecant is in quadrant III. Answer is c.

9. $10 \cdot \frac{180^\circ}{\pi} = 573^\circ - 360^\circ = 213^\circ$. Answer is c.

10. 

   $\csc\frac{5\pi}{3} = \frac{2}{-\sqrt{3}} = \frac{2}{-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$. Answer is a.

11. $\dfrac{\sin 2x}{1+\cos 2x}=\dfrac{2\sin x\cos x}{1+(2\cos^2 x-1)}=\dfrac{2\sin x\cos x}{2\cos^2 x}=\dfrac{\sin x}{\cos x}=\tan x$. Answer is a.

12. $\dfrac{\sin 6x}{\sin 3x}=\dfrac{2\sin 3x\cos 3x}{\sin 3x}=2\cos 3x$. Answer is b.

13. Reminder: $\sin 2x=2\sin x\cos x$, therefore, $\cos\dfrac{x}{3}\sin\dfrac{x}{3}=\dfrac{1}{2}\sin\dfrac{2x}{3}$. Answer is a.

14. $\sin^2 x+\cos^2 x+\cot^2 x=1+\cot^2 x=\csc^2 x$. Answer is a.

15. $-1-\cos^2 x+\sin^2 x=-\cos^2 x-\cos^2 x=-2\cos^2 x$. Answer is d.

16. $\cos(2x+\pi)=\cos 2x\cos\pi-\sin 2x\sin\pi=\cos 2x(-1)-\sin 2x(0)=-\cos 2x$
$\cos 2x=2\cos^2 x-1=1-2\sin^2 x=\cos^2 x-\sin^2 x$ therefore $-\cos 2x$ could not equal b. Answer is b.

17. $\tan x$ is undefined at $\dfrac{\pi}{2}$ and $\dfrac{3\pi}{2}$, $0\le x<2\pi$, at these terminal values $\sin x\ne 0$, $\cos x=0$ Answer is c.

18. $\cot\dfrac{17\pi}{6}=\cot\left(\dfrac{17\pi}{6}-2\pi\right)=\cot\dfrac{5\pi}{6}$ in Quadrant II cotangent is negative

Therefore, reference angle is $-\cot\dfrac{\pi}{6}=-\dfrac{\sqrt{3}}{1}=-\sqrt{3}$ Answer is a.

19. $\sec\theta<0$ in Quadrant II, III, $\tan\theta>0$ in Quadrant I, III, therefore Quadrant III. Answer is c.

20. $\cos 5x\sin 3x+\sin 5x\cos 3x=\sin(5x+3x)=\sin 8x$ Answer is d.

21. $f(x)=-3\sin(\pi+x)=-3\sin(\pi+0)=-3\sin\pi=-3(0)=0$ Answer is b.

22. Positive angle is $-\dfrac{7\pi}{6}=2\pi-\dfrac{7\pi}{6}=\dfrac{5\pi}{6}$, reference angle is $\pi-\dfrac{5\pi}{6}=\dfrac{\pi}{6}$ Answer is b.

23. Graph $y=2^x-\dfrac{1}{\tan x}$ on calculator and find zeros. Answer is a.

24. $\sec\theta=\dfrac{\text{hypotenuse}}{\text{adjacent}}=\dfrac{1}{a}$ Answer is c.

25. Graph $y=\dfrac{1}{\tan x}-\sin 3x$, find crossings of x-axis (zeroes). Answer is b.

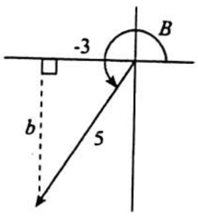
26. Amplitude $|-a|=a$, displacement $=-b$ therefore maximum value $a-b$. Answer is c.

27. Start at vertical displacement value of $-b$ then go down a. Answer is a.

28. $\log\cot x = \log\left(\frac{1}{\tan x}\right) = \log 1 - \log\tan x = -\log\tan x = -a$. Answer is c.

29. $\frac{\cos^3 A - \cos A}{\sin^3 A} = \frac{\cos A(\cos^2 A - 1)}{\sin^3 A} = \frac{-\cos A\cdot\sin^2 A}{\sin^3 A} = \frac{-\cos A}{\sin A} = -\cot A$. Answer is b.

30.



by Pythagoras Theorem, $(-3)^2 + b^2 = 5^2 \rightarrow b = -4$, $\tan B = \frac{-4}{-3} = \frac{4}{3}$. Answer is d.

31. $\frac{\csc^2 x + \sec^2 x}{\csc^2 x - \sec^2 x} = \frac{\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}} = \frac{\frac{\cos^2 x + \sin^2 x}{\cos^2 x\sin^2 x}}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x\sin^2 x}} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x} = \frac{1}{\cos 2x} = \sec 2x$. Answer is c.

32. $\frac{\csc x}{\cos x} = \frac{1}{\sin x\cdot\cos x}$, denominator must not be zero. Answer is d.

33. $\sec x < 0$ in quadrants II, III, therefore, by special angles answers are $\frac{5\pi}{6}, \frac{7\pi}{6}$. Answer is c.

34. Reminder: $\cos(A+B) = \cos A\cos B - \sin A\sin B$, therefore, $\cos\frac{x}{4}\cos\frac{x}{3} - \sin\frac{x}{4}\sin\frac{x}{3} = \cos\left(\frac{x}{4}+\frac{x}{3}\right) =$ $\cos\frac{7x}{12}$. Answer is d

35. $\cos^2 x = \frac{1}{a} \rightarrow \cos x = \pm\frac{1}{\sqrt{a}}$; Reminder: Each trig. function is positive in 2 quadrants and negative in 2 quadrants. Answer is d.

36. $(a\sin\theta - a)(\tan^3\theta - b) = 0 \rightarrow \sin\theta = 1$ or $\tan\theta = \sqrt[3]{b}$, $\sin\theta = 1$ at $\frac{\pi}{2}$, one solution, tangent is positive in quadrants I and III, therefore, 3 total solutions. Answer is b.

37. $(2\sin 3x - 1)(\cos 2x + 1) = 0 \rightarrow \sin 3x = \frac{1}{2}$ or $\cos 2x = -1$

$\sin 3x = \frac{1}{2}$. Sine has a value of $\frac{1}{2}$ in quadrants III, IV. Since the period is $\frac{2\pi}{3}$, to go to 2π requires multiplying $\frac{2\pi}{3}$ by 3, therefore, $2\times 3 = 6$ solutions.

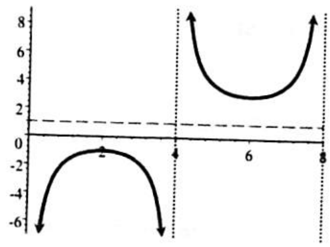
$\cos 2x = -1$. Cosine has a value of -1 only at π. Since the period is $\frac{2\pi}{2} = \pi$, to go to 2π requires multiplying π by 2, therefore, $1\times 2 = 2$ solutions.
Total of 8 solutions. Answer is c.

38. $\dfrac{1+\sec x}{\tan x+\sin x}=\dfrac{1+\dfrac{1}{\cos x}}{\dfrac{\sin x}{\cos x}+\sin x}=\dfrac{\dfrac{\cos x+1}{\cos x}}{\dfrac{\sin x+\sin x\cos x}{\cos x}}=\dfrac{\cos x+1}{\sin x(1+\cos x)}=\dfrac{1}{\sin x}=\csc x$. Answer is a.

39. $\dfrac{\cos x}{1-\sin x}-\dfrac{1}{\cos x}=\dfrac{\cos^2 x-1+\sin x}{\cos x(1-\sin x)}=\dfrac{-\sin^2 x+\sin x}{\cos x(1-\sin x)}=\dfrac{\sin x(1-\sin x)}{\cos x(1-\sin x)}=\tan x$. Answer is a.

40. A cosecant graph is a reciprocal of the sine graph, therefore, where sine equals zero, cosecant is undefined.

For $y=-2\csc\frac{\pi}{4}x+1$, *Period* $=\dfrac{2\pi}{\frac{\pi}{4}}=8$, *Amplitude of sine*: $|-2|=2$, *Vertical displacement* = 1.



- Graph is undefined when $x=0$, 4, 8.
- Graph has a maximum of -1 when $x=2$, and a minimum of 3 when $x=6$
- Therefore, the range for $4\le x<6$ is $y>3$.

Answer is c

41. Cosine equals zero at $\frac{\pi}{2}+n\pi$, therefore secant, the reciprocal of cosine, is undefined at $\frac{\pi}{2}+n\pi$.

Then $\frac{\pi}{6}(x+1)\ne\frac{\pi}{2}+n\pi \;\rightarrow\; x+1\ne 3+6n \;\rightarrow\; x\ne 2+6n$. Answer is d.

42. $\sin 2x=1$, sine equals 1 at $\frac{\pi}{2}$, therefore, $2x=\frac{\pi}{2}+2n\pi \;\rightarrow\; x=\frac{\pi}{4}+n\pi$, n an integer. Answer is a.

43. By the double angle identity, $y=1-2\sin^2 6x = \cos 12x$, $P=\frac{2\pi}{12}=\frac{\pi}{6}$. Answer is a.

44. $\dfrac{\csc^2 x-1}{\sec^2 x-1}=\dfrac{\dfrac{1}{\sin^2 x}-1}{\dfrac{1}{\cos^2 x}-1}=\dfrac{\dfrac{1-\sin^2 x}{\sin^2 x}}{\dfrac{1-\cos^2 x}{\cos^2 x}}=\dfrac{\cos^2 x}{\sin^2 x}\cdot\dfrac{\cos^2 x}{\sin^2 x}=\dfrac{\cos^4 x}{\sin^4 x}=\cot^4 x$. Answer is d.

45. Set calculator in radian mode, set window x: min 0, x: max 60, y: min 0, y: max 55

Graph $y=25\sin\frac{\pi}{30}(x-10)+76$ and $y=40$ and find intersection points

(34.52 , 40) and (15.68 , 40) , difference $34.52-15.68=18.64$. Answer is b.

46. $\dfrac{\cot x}{2+3\cos x}=\dfrac{\cos x}{\sin x(2+3\cos x)}$, therefore, restrictions are $\sin x\ne 0$, $\cos x\ne-\frac{2}{3}$ Answer is d.

47. $a\sin x + b = 0 \rightarrow \sin x = -\frac{b}{a}$, since $0 < a < b$, $\frac{b}{a} > 1$, therefore no solutions

$a\tan x + a = 0 \rightarrow \tan x = 1$, therefore 2 solutions

$a\sec x - b = 0 \rightarrow \sec x = \frac{b}{a}$ since $0 < a < b$, $\frac{b}{a} > 1$, therefore 2 solutions

for a total of 4 solutions. Answer is b.

48. The vertical displacement is b, therefore, the graph is $\geq b + a$ and $\leq b - a$ Answer is a.

49. $8\sin^2 6x - 4 = -4(1 - 2\sin^2 6x) = -4\cos 2(6x) = -4\cos 12x$ Answer is c.

50. Period $= \frac{2\pi}{|b|}$, $b > 0 \rightarrow P = \frac{2\pi}{b}$ Therefore minimum value occurs $\frac{1}{2}$ a period away at $\frac{\pi}{b}$ Answer is c.

51. $\sin A\cos B + \cos A\sin B = \sin(A + B) = \sin(90^\circ - B + B) = \sin 90^\circ = 1$ Answer is b.

52. $\sec(\pi - x) = \frac{1}{\cos(\pi - x)} = \frac{1}{\cos\pi\cos x + \sin\pi\sin x} = \frac{1}{-\cos x} = -\sec x$ Answer is a.

53. $y = -4\sin 6x\cos 6x = -2\sin 2(6x) = -2\sin 12x$, amplitude $|-2| = 2$ therefore range is $-2 \leq y \leq 2$

Answer is a.

54. $$\frac{\csc^2 x + \sec^2 x}{\csc^2 x - \sec^2 x} = \frac{\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}} = \frac{\frac{\cos^2 x + \sin^2 x}{\sin^2 x\cos^2 x}}{\frac{\cos^2 x - \sin^2 x}{\sin^2 x\cos^2 x}} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x} = \frac{1}{\cos 2x} = \sec 2x.$$ Answer is d.

55. $\csc x = -1.325$, determine reference angle $\sin x = \frac{1}{1.325} \rightarrow x = \sin^{-1}\left(\frac{1}{1.325}\right) = 0.855$

cosecant is negative in quadrant III, IV therefore $2\pi - 0.855 = 5.43$ and $\pi + 0.855 = 4.00$ Answer is d.

56. Remember: $\cos(A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

Therefore, $\cos\left(\frac{\pi}{6} + x\right) \cdot \cos\left(\frac{\pi}{6} - x\right) - \sin\left(\frac{\pi}{6} + x\right) \cdot \sin\left(\frac{\pi}{6} - x\right) = \cos\left[\left(\frac{\pi}{6} + x\right) + \left(\frac{\pi}{6} - x\right)\right] = \cos\frac{\pi}{3} = \frac{1}{2}$,

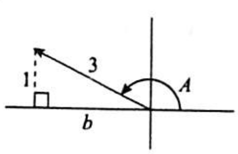
Answer is b.

57. $y = 6\sin x\cos^3 x + 6\sin^3 x\cos x = 6\sin x\cos x(\cos^2 + \sin^2 x) = 6\sin x\cos x = 3\sin 2x$

$A = 3$, $B = 2$ Answer is c.

58. $\frac{\sin A}{1 + \cos A} - \frac{1 - \cos A}{\sin A} = \frac{\sin^2 A - 1 + \cos^2 A}{\sin A(1 + \cos A)} = \frac{1 - 1}{\sin A(1 + \cos A)} = 0$ Answer is a.

59.



$b^2 + 1^2 = 3^2 \rightarrow b = -\sqrt{8} = -2\sqrt{2}$, then $\cos A = \frac{-2\sqrt{2}}{3}$

Therefore, $\sin 2A = 2\sin A\cos A = 2\left(\frac{1}{3}\right)\left(\frac{-2\sqrt{2}}{3}\right) = \frac{-4\sqrt{2}}{9}$. Answer is c.

60. $\sin A = \frac{-3}{5}$, therefore, by Pythagoras Theorem, $\cos A = \frac{4}{5}$ in quadrant IV

$\cos B = \frac{3}{5}$, therefore, by Pythagoras Theorem, $\sin B = \frac{-4}{5}$ in quadrant IV

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B = \left(\frac{-3}{5}\right)\left(\frac{3}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{-4}{5}\right) = \frac{-9}{25} - \frac{16}{25} = -1$ Answer is a

61. Tangent has a value of $-\sqrt{3}$ at $\frac{2\pi}{3}$ and $\frac{5\pi}{3}$ which is π distance apart.

Therefore, $bx = \frac{2\pi}{3} + n\pi \rightarrow x = \frac{2\pi}{3b} + \frac{n\pi}{b}$. Answer is b.

62. $\cos\left(x+\frac{\pi}{2}\right) - \cos\left(x-\frac{\pi}{2}\right) = \left(\cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2}\right) - \left(\cos x \cos\frac{\pi}{2} + \sin x \sin\frac{\pi}{2}\right)$

$= (0 - \sin x) - (0 + \sin x)$

$= -2\sin x$. Answer is b.

63. $8\sin^4 x + 2\sin^2 x - 1 = 0 \rightarrow (4\sin^2 x - 1)(2\sin^2 x + 1) = 0 \rightarrow \sin x = \pm\frac{1}{2}$, $\sin x = \pm\sqrt{\frac{-1}{2}}$

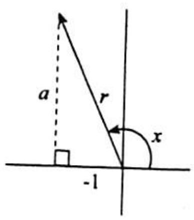
$\sin x = \pm\frac{1}{2}$ at 30°, 150°, 210°, 330°, $\sin x = \pm\sqrt{\frac{-1}{2}}$, ϕ. Answer is c.

64. $(\sin x - \cos x)^2 - (\sin x + \cos x)^2 = (\sin^2 x - 2\sin x\cos x + \cos^2 x) - (\sin x^2 + 2\sin x\cos x + \cos^2 x)$

$= 1 - 2\sin x\cos x - 1 - 2\sin x\cos x = -4\sin x\cos x = -2\sin 2x$.

Answer is d.

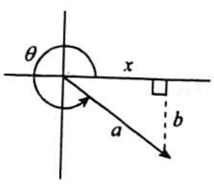
65. Tangent negative in quadrant II and IV, cosine negative in quadrant II and III, therefore, quadrant II



$r^2 = a^2 + (-1)^2$

$r = \sqrt{a^2+1}$ $\csc x = \frac{\sqrt{a^2+1}}{a}$ Answer is b.

66. $\tan\theta < 0$ in quadrant II and IV , $\sec\theta > 0$ in quadrant I and IV, therefore, quadrant IV



$a^2 = x^2 + b^2$

$x^2 = a^2 - b^2$

$x = \sqrt{a^2 - b^2}$ $\cos\theta = \frac{\sqrt{a^2-b^2}}{a}$ Answer is a.

67. cosine has value $-\frac{1}{2}$ at $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$, therefore, $2x = \frac{2\pi}{3} + 2n\pi$, $2x = \frac{4\pi}{3} + 2n\pi$

$x = \frac{\pi}{3} + n\pi$, $x = \frac{2\pi}{3} + n\pi$ Answer is b.

68. $\cos x + 2\cos^2 x = 0 \to \cos x(1+2\cos x) = 0 \to \cos x = 0,\ -\frac{1}{2}$

$\cos x = 0$ at $\frac{\pi}{2} + n\pi$, $\cos x = -\frac{1}{2}$ at $\frac{2\pi}{3} + 2n\pi$ and $\frac{4\pi}{3} + 2n\pi$ Answer is b.

69. If the maximum point is $(2, 16)$ and the nearest minimum point is $(7, 4)$, then its half period is 5 and the period is 10; a vertical displacement of $\frac{4+16}{2} = 10$; and an amplitude of $\frac{16-4}{2} = 6$, therefore,

$p = \frac{2\pi}{b} = 10 \to b = \frac{\pi}{5}$, phase shift = 2

So $y = a\cos b(x-c) + d \to y = 6\cos\frac{\pi}{5}(x-2) + 10$ Answer is a

70. All graphs except 'a' are reciprocal graphs of the diagram. Answer is a.

71. Amplitude is 25, $p = \frac{2\pi}{b} = 80 \to b = \frac{\pi}{40}$, vertical displacement is $25 + 2 = 27$, start upside down from standard cosine, therefore, $h = -25\cos\left(\frac{\pi}{40}t\right) + 27$ Answer is c.

72. Set calculator in radian mode, set window x: min 0, x: max 60, y: min 0, y: max 55

Graph $y = -25\cos\left(\frac{\pi}{40}x\right) + 27$ and $y = 35$, and determine the 1st intersecting point. Answer is c.

73. If the maximum depth occurred at 4:00 and the minimum depth occurred 6 hours later then the period is 12 hours. Therefore $P = \frac{2\pi}{b} \to 12 = \frac{2\pi}{b} \to b = \frac{\pi}{6}$. If the maximum depth is 8 meters and minimum depth 2 hours then the displacement is 5 meters with amplitude 3 meters.

The only equation that fit these parameters is $d(t) = 3\sin\frac{\pi}{6}(x-1) + 5$. Answer is c.

74. Graph $y = 3\sin\frac{\pi}{6}(x-1) + 5$ set windows $x[0, 13]$, $y[-1, 10]$

On calculator enter $x = 12$, y value is 3.5 or

$y = 3\sin\frac{\pi}{6}(12-1) + 5 = 3\sin\frac{11\pi}{6} + 5 = 3\left(-\frac{1}{2}\right) + 5 = 3.5$ Answer is c.

75. Graph $y = 3\sin\frac{\pi}{6}(x-1) + 5$ and $y = 7$ and find first intersection $x = 2.39$ hr $\to 2:24$ a.m.

Answer is a.