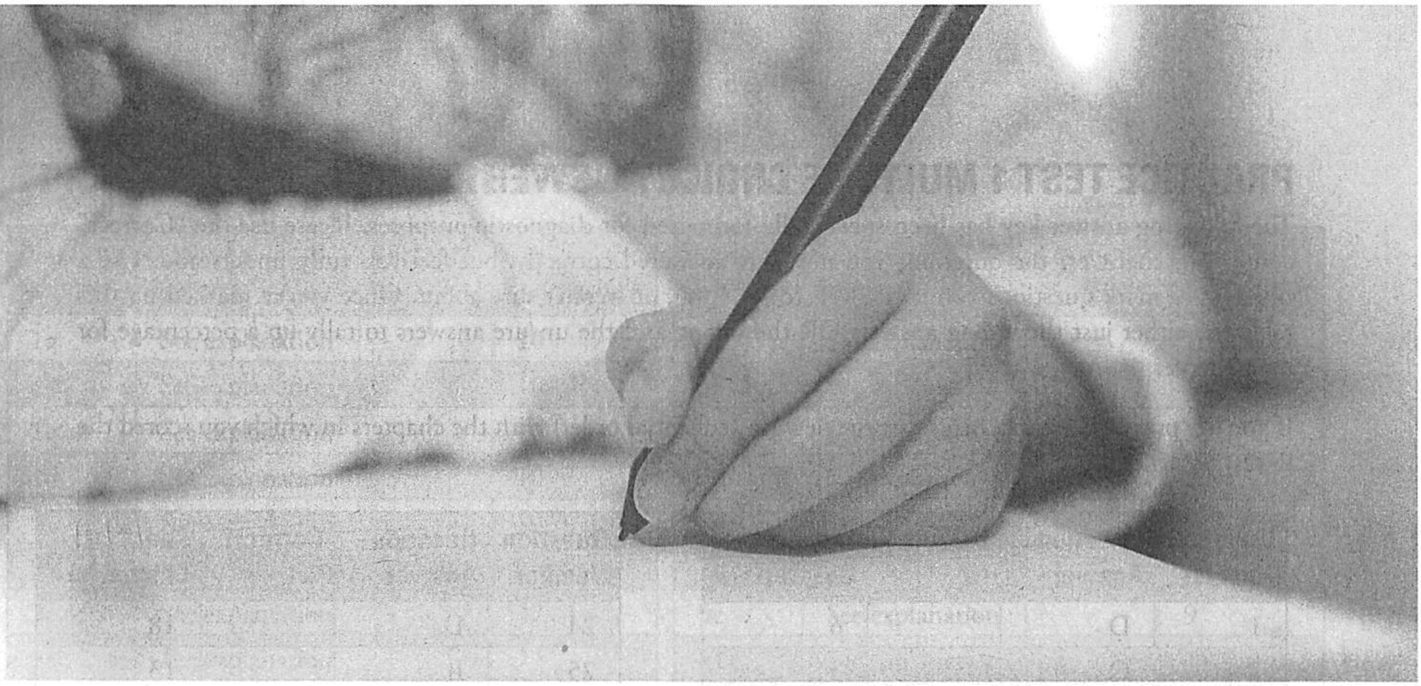


Practice Test 1: Answers and Explanations

PRACTICE TEST 1 MULTIPLE CHOICE ANSWER KEY

The following answer key has been specifically formatted for diagnostic purposes. Please use the "Correct" column to check off the questions you not only answered correctly, but feel you fully understood. Use a squiggle to mark questions you used POE to guess on, or weren't sure about. Once you've marked up this table use either just the wrong answers OR the wrong and the unsure answers to tally up a percentage for each chapter.

If you are pressed for time, begin your review (in sequential order) with the chapters in which you scored the lowest percentage.

Question Number	Question Answer	Correct? ✎	See Chapter(s) #
1	D		6
2	C		3
3	C		6
4	C		3, 12
5	B		7
6	C		14
7	B		6
8	D		8
9	A		13
10	A		16
11	B		N/A*
12	A		13
13	A		8
14	C		N/A*
15	B		4, 5
16	A		9
17	D		10
18	C		16
19	D		13
20	B		6
21	A		11, 15
22	D		17
23	B		3, 12

Question Number	Question Answer	Correct? ✎	See Chapter(s) #
24	D		18
25	B		13
26	A		6
27	B		13
28	D		6
29	B		3, 12
30	A		13, 15
31	C		13
32	D		13
33	C		13
34	B		6
35	A		7
36	C		16
37	A		14
38	D		13
39	A		9
40	A		10
41	A		14
42	C		13, 15
43	B		6
44	D		10
45	B		10

*A small percentage of questions on the AP Calculus AB Exam pull from Pre-Calculus topics. This book does not re-cover that material, so there is no chapter reference. Do, however, refer to the explanation for more details.

PRACTICE TEST 1 FREE-RESPONSE ANSWER KEY

Question Number	Question Answer	Correct? ✎	See Chapter(s) #
1a	See explanation		10
1b	See explanation		10
1c	See explanation		10
1d	See explanation		10
2a	See explanation		16
2b	See explanation		17
2c	See explanation		17
3a	See explanation		7
3b	See explanation		8
3c	See explanation		7

Question Number	Question Answer	Correct? ✎	See Chapter(s) #
4a	See explanation		10
4b	See explanation		10
4c	See explanation		10
4d	See explanation		10
5a	See explanation		8
5b	See explanation		9
5c	See explanation		9
6a	See explanation		14
6b	See explanation		14
6c	See explanation		5

CHAPTER 3 TEST SCORE SELF-EVALUATION

\# WRONG QUESTIONS

= %

4

YOUR SCORE

\# TOTAL QUESTIONS

CHAPTER 4 TEST SCORE SELF-EVALUATION

\# WRONG QUESTIONS

= %

1

YOUR SCORE

\# TOTAL QUESTIONS

CHAPTER 5 TEST SCORE SELF-EVALUATION

\# WRONG QUESTIONS

= %

2

YOUR SCORE

\# TOTAL QUESTIONS

CHAPTER 6 TEST SCORE SELF-EVALUATION

\# WRONG QUESTIONS

= %

8

YOUR SCORE

\# TOTAL QUESTIONS

CHAPTER 7 TEST SCORE SELF-EVALUATION

\# WRONG QUESTIONS

= %

4

YOUR SCORE

\# TOTAL QUESTIONS

CHAPTER 8 TEST SCORE SELF-EVALUATION

\# WRONG QUESTIONS

= %

4

YOUR SCORE

\# TOTAL QUESTIONS

CHAPTER 9 TEST SCORE SELF-EVALUATION

WRONG QUESTIONS / 4 # TOTAL QUESTIONS = ____ % YOUR SCORE

CHAPTER 10 TEST SCORE SELF-EVALUATION

WRONG QUESTIONS / 14 # TOTAL QUESTIONS = ____ % YOUR SCORE

CHAPTER 11 TEST SCORE SELF-EVALUATION

WRONG QUESTIONS / 1 # TOTAL QUESTIONS = ____ % YOUR SCORE

CHAPTER 12 TEST SCORE SELF-EVALUATION

WRONG QUESTIONS / 3 # TOTAL QUESTIONS = ____ % YOUR SCORE

CHAPTER 13 TEST SCORE SELF-EVALUATION

WRONG QUESTIONS / 11 # TOTAL QUESTIONS = ____ % YOUR SCORE

CHAPTER 14 TEST SCORE SELF-EVALUATION

WRONG QUESTIONS / 5 # TOTAL QUESTIONS = ____ % YOUR SCORE

CHAPTER 15 TEST SCORE SELF-EVALUATION

WRONG QUESTIONS / 3 # TOTAL QUESTIONS = ____ % YOUR SCORE

CHAPTER 16 TEST SCORE SELF-EVALUATION

WRONG QUESTIONS / 14 # TOTAL QUESTIONS = ____ % YOUR SCORE

CHAPTER 17 TEST SCORE SELF-EVALUATION

WRONG QUESTIONS / 3 # TOTAL QUESTIONS = ____ % YOUR SCORE

CHAPTER 18 TEST SCORE SELF-EVALUATION

WRONG QUESTIONS / 1 # TOTAL QUESTIONS = ____ % YOUR SCORE

ANSWERS AND EXPLANATIONS TO SECTION I

1. **D** Take the derivative: $f'(x) = 6x - 10 + \frac{1}{x-3}$ and plug in $x = 4$.

$$f'(4) = 6(4) - 10 + \frac{1}{(4)-3} = 15$$

2. **C** **Step 1:** To solve this problem, you need to remember how to evaluate limits. Always do limit problems on the first pass. Whenever we have a limit of a polynomial fraction where $x \to \infty$, divide the numerator and the denominator, separately, by the highest power of x in the fraction.

$$\lim_{x\to\infty} \frac{5x^2 - 3x + 1}{4x^2 + 2x + 5} = \lim_{x\to\infty} \frac{\frac{5x^2}{x^2} - \frac{3x}{x^2} + \frac{1}{x^2}}{\frac{4x^2}{x^2} + \frac{2x}{x^2} + \frac{5}{x^2}}$$

Step 2: Simplify $\lim_{x\to\infty} \frac{5 - \frac{3}{x} + \frac{1}{x^2}}{4 + \frac{2}{x} + \frac{5}{x^2}}$.

Step 3: Now take the limit. Remember that the $\lim_{x\to\infty} \frac{k}{x^n} = 0$, if $n > 0$, where k is a constant. Thus, we get

$$\lim_{x\to\infty} \frac{5 - \frac{3}{x} + \frac{1}{x^2}}{4 + \frac{2}{x} + \frac{5}{x^2}} = \lim_{x\to\infty} \frac{5 - 0 + 0}{4 + 0 + 0} = \frac{5}{4}$$

3. **C** **Step 1:** We need to use the Quotient Rule to evaluate this derivative. Remember, the derivative of $\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$. But, before we take the derivative, we should factor an x out of the top and bottom and cancel, simplifying the quotient.

$$f(x) = \frac{3x^2 + x}{3x^2 - x} = \frac{x(3x+1)}{x(3x-1)} = \frac{3x+1}{3x-1}$$

Step 2: Now take the derivative.

$$f'(x) = \frac{(3x-1)(3) - (3x+1)(3)}{(3x-1)^2}$$

Step 3: Simplify.

$$\frac{9x - 3 - 9x - 3}{(3x-1)^2} = \frac{-6}{(3x-1)^2}$$

4. **C** We know that $\lim_{x\to 0}\frac{\sin(ax)}{x} = a$, so we get 3(2) = 6. Or we could get the answer using L'Hôpital's Rule, because the limit is of an indeterminate form: $\frac{0}{0}$. We take the derivative of the top and bottom and get: $\lim_{x\to 0}\frac{3\sin(2x)}{x} = \lim_{x\to 0}\frac{3(2)\cos(2x)}{1} = 3(2) = 6$.

5. **B** We need to use Implicit Differentiation to find $\frac{dy}{dx}$. First, take the derivative with respect to x of both sides: $\frac{dy}{dx} = \cos(xy)\left(x\frac{dy}{dx} + y\right)$. Remember that you need to use the Product Rule to find the derivative of xy.

   Now distribute on the right side: $\frac{dy}{dx} = x\frac{dy}{dx}\cos(xy) + y\cos(xy)$.

   Next, group the terms containing $\frac{dy}{dx}$ on the left side: $\frac{dy}{dx} - x\frac{dy}{dx}\cos(xy) = y\cos(xy)$.

   Factor out $\frac{dy}{dx}$: $x\frac{dy}{dx}(1 - x\cos(xy)) = y\cos(xy)$.

   And divide to isolate $\frac{dy}{dx}$: $\frac{dy}{dx} = \frac{y\cos(xy)}{1 - x\cos(xy)}$.

6. **C** The right Riemann sum will use the values on the right hand sides of the intervals [0, 1], [1, 2], [2, 3], and [3, 4]. We get: $f(1) + f(2) + f(3) + f(4) = 4 + 13 + 28 + 49 = 94$.

7. **B** First, differentiate h using the Product Rule: $h'(x) = f'(x)g(4x) + 4f(x)g'(4x)$. Now substitute $x = 2$ and get: $h'(2) = f'(2)g(8) + 4f(2)g'(8)$.

8. **D** Here we do everything that we normally do for finding the equations of tangent lines, except that we use the negative reciprocal of the slope to find the normal line. This is because the normal line is perpendicular to the tangent line.

   Step 1: First, find the slope of the tangent line.

$$\frac{dy}{dx} = \frac{1}{2}(3x^2 + 2x)^{-\frac{1}{2}}(6x + 2)$$

   Step 2: DON'T SIMPLIFY. Immediately plug in $x = 2$. We get

$$\frac{dy}{dx} = \frac{1}{2}(3x^2 + 2x)^{-\frac{1}{2}}(6x + 2) = \frac{1}{2}(3(2)^2 + 2(2))^{-\frac{1}{2}}(6(2) + 2) = \frac{1}{2}(16)^{-\frac{1}{2}}(14) = \frac{7}{4}$$

   This means that the slope of the tangent line at $x = 2$ is $\frac{7}{4}$, so the slope of the normal line is $-\frac{4}{7}$.

   Step 3: Then the equation of the normal line is $(y - 4) = -\frac{4}{7}(x - 2)$.

Step 4: Multiply through by 7 and simplify.

$$7y - 28 = -4x + 8$$
$$4x + 7y = 36$$

9. **A** The derivative of cot x is $-\csc^2 x$, so $4\int \csc^2 x\, dx = -4\cot x + C$.

10. **A** This problem is just asking us to find a higher order derivative of a trigonometric function.

Step 1: The first derivative requires the Chain Rule.

$f(x) = \cos^2 x$
$f'(x) = 2(\cos x)(-\sin x) = -2\cos x \sin x$

Step 2: The second derivative requires the Product Rule.

$f'(x) = -2\cos x \sin x$
$f''(x) = -2(\cos x \cos x - \sin x \sin x) = -2(\cos^2 x - \sin^2 x)$

Step 3: Now plug in π for x and simplify.

$-2(\cos^2(\pi) - \sin^2(\pi)) = -2(1 - 0) = -2$

11. **B** **Step 1:** To find $g(f(x))$, all you need to do is to replace all of the x's in $g(x)$ with $f(x)$'s.

$$g(f(x)) = 3f(x) = 3\left(\frac{5}{x^2+1}\right) = \frac{15}{x^2+1}$$

Step 2: Now all we have to do is plug in 2 for x.

$$g(f(2)) = \frac{15}{2^2+1} = 3$$

12. **A** We can evaluate this integral using U-substitution. Let $u = 1 + x^2$ and $du = 2xdx$, $\frac{1}{2}du = xdx$. Substituting into the integrand, we get: $\frac{1}{2}\int\sqrt{u}\,du$. Evaluate the integral: $\frac{1}{2}\int\sqrt{u}\,du = \frac{1}{2}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{3}u^{\frac{3}{2}} + C$. And substitute back: $\frac{1}{3}\left(1+x^2\right)^{\frac{3}{2}} + C$.

13. **A** This is another equation of a tangent line problem, combined with implicit differentiation. Often, the AP Exam has more than one tangent line problem, so make sure that you can do these well!

By the way, do you remember the derivative of $\ln(f(x))$? It is $\frac{f'(x)}{f(x)}$.

Step 1: First, take the derivative of the equation.

$$6x\frac{dx}{dx} + \frac{5}{y}\frac{dy}{dx} = 0$$

Step 2: Next, we simplify and solve for $\frac{dy}{dx}$.

$$6x + \frac{5}{y}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-6xy}{5}$$

Step 3: Now, we plug in 2 for x and 1 for y to get the slope of the tangent line.

$$\frac{dy}{dx} = \frac{-6(2)(1)}{5} = \frac{-12}{5}$$

14. **C** The AP people expect you to remember a lot of your trigonometry, so if you're rusty, review the unit in the Appendix.

Step 1: In an equation of the form $f(x) = A \sin B(x \pm C) \pm D$, you should know four components. The amplitude of the equation is $|A|$, the horizontal or phase shift is $\pm C$, the vertical shift is $\pm D$, and the fundamental period is $\frac{2\pi}{B}$.

The same is true for $f(x) = A \cos B(x \pm C) \pm D$.

Step 2: All we have to do is plug into the formula for the period.

$$\frac{2\pi}{B} = \frac{2\pi}{\frac{\pi}{4}} = 8$$

15. **B** f is not continuous at a and d. At b, the function is continuous but it has a cusp, which will have a vertical tangent line and so not be differentiable.

16. **A** This problem requires you to know how to find maxima/minima. This is a part of curve sketching and is one of the most important parts of differential calculus. A function has *critical points* where the derivative is zero or undefined (which is never a problem when the function is an ordinary polynomial). After finding the critical points, test them to determine whether they are maxima or minima or something else.

Step 1: First, as usual, take the derivative and set it equal to zero.

$$f'(x) = 3x^2 - 18x - 120$$

$$3x^2 - 18x - 120 = 0$$

Step 2: Find the values of x that make the derivative equal to zero. These are the critical points.

$$3x^2 - 18x - 120 = 0$$
$$x^2 - 6x - 40 = 0$$
$$(x - 10)(x + 4) = 0$$
$$x = \{10, -4\}$$

Step 3: In order to determine whether a critical point is a maximum or a minimum, we need to take the second derivative.

$$f''(x) = 6x - 18$$

Step 4: Now, we plug the critical points from Step 2 into the second derivative. If it yields a negative value, then the point is a maximum. If it yields a positive value, then the point is a minimum. If it yields zero, it is neither, and is most likely a point of inflection.

$$6(10) - 18 = 42$$
$$6(-4) - 18 = -42$$

Therefore, 10 is a minimum.

17. D **Step 1:** Because acceleration is the derivative of velocity, if we know the acceleration of a particle, we can find the velocity by integrating the acceleration with respect to t.

$$v = \int a\, dt = \int (4t - 12)\, dt = 2t^2 - 12t + C$$

Next, because the velocity is 10 at $t = 0$, we can plug in 0 for t and solve for the constant.

$$2(0)^2 - 12(0) + C = 10$$

Therefore, $C = 10$ and the velocity, $v(t)$, is $2t^2 - 12t + 10$.

Step 2: In order to find when the particle is changing direction, we need to know when the velocity is equal to zero, so we set $v(t) = 0$ and solve for t.

$$2t^2 - 12t + 10 = 0$$
$$t^2 - 6t + 5 = 0$$
$$(t - 5)(t - 1) = 0$$
$$t = \{1, 5\}$$

Now, provided that the acceleration is not also zero at $t = \{1, 5\}$, the particle will be changing direction at those times. The acceleration is found by differentiating the equation for velocity with respect to time: $a(t) = 4t - 12$. This is not zero at either $t = 1$ or $t = 5$. Therefore, the particle is changing direction when $t = 1$ and $t = 5$.

18. **C** First, find where the two curves intersect. Set them equal to each other and solve:

$$12 - x^2 = 2x^2$$
$$12 = 3x^2$$
$$x^2 = 4$$
$$x = \pm 2$$

Therefore, in order to find the area of the region, we need to evaluate the integral: $\int_{-2}^{2}(12 - x^2) - (2x^2)\,dx$

Simplify the integrand: $\int_{-2}^{2}(12 - x^2) - (2x^2)\,dx = \int_{-2}^{2}(12 - 3x^2)\,dx$.

Use the Power Rule: $\int_{-2}^{2}(12 - 3x^2)\,dx = (12x - x^3)\Big|_{-2}^{2}$.

And evaluate: $(12x - x^3)\Big|_{-2}^{2} = (24 - 8) - (-24 + 8) = 32$.

19. **D** This problem requires that you know your rules of exponential functions.

Step 1: First of all, $e^{3\ln x} = e^{\ln x^3} = x^3$. So we can rewrite the integral as

$$\int (e^{3\ln x} + e^{3x})\,dx = \int (x^3 + e^{3x})\,dx$$

Step 2: The rule for the integral of an exponential function is $\int e^k\,dx = \frac{1}{k}e^{kx} + C$.

Now we can do the integral: $\int (x^3 + e^{3x})\,dx = \frac{x^4}{4} + \frac{1}{3}e^{3x} + C$.

20. **B** This problem is just a complicated derivative, requiring you to be familiar with the Chain Rule and the Product Rule.

Step 1: $f'(x) = \frac{1}{2}(x^3 + 5x + 121)^{-\frac{1}{2}}(3x^2 + 5)(x^2 + x + 11) + (x^3 + 5x + 121)^{\frac{1}{2}}(2x + 1)$.

Step 2: Whenever a problem asks you to find the value of a complicated derivative at a particular point, NEVER simplify the derivative. Immediately plug in the value for x and do arithmetic instead of algebra.

$$f'(0) = \frac{1}{2}(0^3 + 5(0) + 121)^{-\frac{1}{2}}(3(0)^2 + 5)((0)^2 + (0) + 11) + ((0)^3 + 5(0) + 121)^{\frac{1}{2}}(2(0) + 1)$$

$$= \frac{1}{2}(121)^{-\frac{1}{2}}(5)(11) + (121)^{\frac{1}{2}}(1) = \frac{5}{2} + 11 = \frac{27}{2}$$

21. **A** This problem requires you to know how to find the derivative of an exponential function. According to the rule, if a function is of the form $a^{f(x)}$, its derivative is $a^{f(x)}(\ln a)\,f'(x)$. Now all we have to do is follow the rule!

Step 1: $f(x) = 5^{3x}$

$f'(x) = 5^{3x}(\ln 5)(3)$

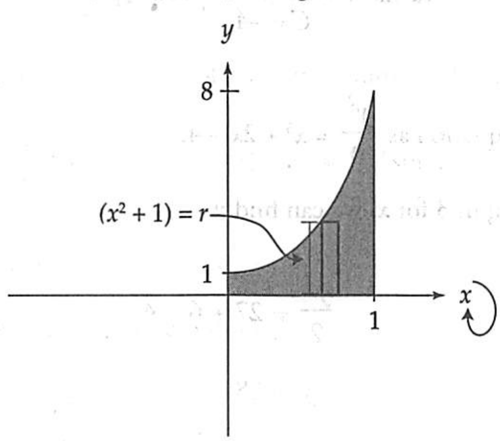
Step 2: If you remember your rules of logarithms, $3\ln 5 = \ln(5^3) = \ln 125$.

So we can rewrite the answer to $f'(x) = 5^{3x}(\ln 5)(3) = 5^{3x}(\ln 125)$.

22. **D** This problem requires you to know how to find the volume of a solid of revolution.

If you have a region between two curves, from $x = a$ to $x = b$, then the volume generated when the region is revolved around the x-axis is: $\pi\int_a^b \left[f(x)^2 - g(x)^2\right] dx$, if $f(x)$ is above $g(x)$ throughout the region.

Step 1: First, we have to determine what the region looks like. The curve looks like the following:



The shaded region is the part that we are interested in. Notice that the curve is always above the x-axis (which is $g(x)$). Now we just follow the formula.

$$\pi\int_0^1\left[\left[(x^2+1)^3\right]^2 - [0]^2\right] dx = \pi\int_0^1 (x^2+1)^6\, dx$$

23. **B** We can get the answer using L'Hôpital's Rule, because the limit is of an indeterminate form: $\frac{0}{0}$. We take the derivative of the top and bottom and get: $\lim\limits_{x\to 0}\dfrac{\sin\left(\frac{\pi}{2}+x\right)-1}{x} = \lim\limits_{x\to 0}\dfrac{\cos\left(\frac{\pi}{2}+x\right)}{1} = \cos\frac{\pi}{2} = 0$.

Now break this into limits that we can easily evaluate.

24. **D** This is a very basic differential equation. See page 344 for a discussion of *separation of variables*.

Step 1: First, separate the variables. Then, we get

$$y\, dy = (3x^2 + 2)\, dx$$

Step 2: Now integrate both sides.

$$\int y\,dy = \int (3x^2 + 2)\,dx$$

$$\frac{y^2}{2} = x^3 + 2x + C$$

Notice how we use only one constant. All we have to do now is solve for C. We do this by plugging in 2 for x and 4 for y.

$$\frac{16}{2} = 2^3 + 4 + C$$

$$C = -4$$

So we can rewrite the equation as $\frac{y^2}{2} = x^3 + 2x - 4$.

Step 3: Now, if we plug in 3 for x, we can find y.

$$\frac{y^2}{2} = 27 + 6 - 4$$

$$y^2 = 58$$

$$y = \pm\sqrt{58}$$

25. **B** This is another inverse trigonometric integral.

Step 1: We know that $\int \frac{dx}{1 + x^2} = \tan^{-1}(x) + C$.

(See problem 9 if you're not sure of this.) The trick here is to get the denominator of the fraction in the integrand to be of the correct form. If we factor 9 out of the denominator, we get

$$\int \frac{dx}{9 + x^2} = \int \frac{dx}{9\left(1 + \frac{x^2}{9}\right)} = \frac{1}{9}\int \frac{dx}{1 + \frac{x^2}{9}} = \frac{1}{9}\int \frac{dx}{1 + \left(\frac{x}{3}\right)^2}$$

Step 2: Now use u-substitution to evaluate this integral.

Let $u = \frac{x}{3}$ and $du = \frac{1}{3}\,dx$ or $3\,du = dx$. Then we have

$$\frac{1}{9}\int \frac{dx}{1 + \left(\frac{x}{3}\right)^2} = \frac{1}{9}\int \frac{3\,du}{1 + u^2} = \frac{1}{3}\int \frac{du}{1 + u^2} = \frac{1}{3}\tan^{-1}(u) + C$$

Step 3: Now all we have to do is reverse the u-substitution and we're done.

$$\frac{1}{3}\tan^{-1}(u) + C = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

26. A Think of $\cos^3(x + 1)$ as $[\cos(x + 1)]^3$.

Step 1: First, we take the derivative of the outside function and ignore the inside functions. The derivative of u^3 is $3u^2$. We get $\frac{d}{dx}[u]^3 = 3[u]^2$.

Step 2: Next, take the derivative of the cosine term and multiply. The derivative of $\cos u$ is $-\sin u$.

$$\frac{d}{dx}[\cos(u)]^3 = -3[\cos(u)]^2 \sin(u)$$

Step 3: Finally, we take the derivative of $x + 1$ and multiply. The derivative of $x + 1$ is 1.

$$\frac{d}{dx}[\cos(x + 1)]^3 = -3[\cos(x + 1)]^2 \sin(x + 1)$$

27. B We can do this integral with u-substitution.

Step 1: Let $u = x + 3$. Then $du = dx$ and $u - 3 = x$.

Step 2: Substituting, we get

$$\int x\sqrt{x+3}\, dx = \int (u-3)u^{\frac{1}{2}}\, du$$

Why is this better than the original integral, you might ask? Because now we can distribute and the integral becomes easy.

Step 3: When we distribute, we get

$$\int (u-3)u^{\frac{1}{2}}\, du = \int\left(u^{\frac{3}{2}} - 3u^{\frac{1}{2}}\right) du$$

Step 4: Now we can integrate.

$$\int\left(u^{\frac{3}{2}} - 3u^{\frac{1}{2}}\right) du = \frac{2}{5}u^{\frac{5}{2}} - 3\left(\frac{2}{3}u^{\frac{3}{2}}\right) + C$$

Step 5: Substituting back, we get

$$\frac{2}{5}u^{\frac{5}{2}} - 3\left(\frac{2}{3}u^{\frac{3}{2}}\right) + C = \frac{2}{5}(x+3)^{\frac{5}{2}} - 2(x+3)^{\frac{3}{2}} + C$$

28. **D** Here, we use the Chain Rule.

Step 1: First, take the derivative of the outside function.

The derivative of ln u is $\frac{du}{u}$.

We get

$$\frac{d}{dx}\ln(\ln(u)) = \frac{1}{\ln(u)}$$

Step 2: Now we take the derivative of the function in the denominator. Once again, the function is ln u.

We get

$$\frac{d}{dx}\ln(\ln(1-x)) = \frac{1}{\ln(1-x)}\left(\frac{-1}{1-x}\right) = \frac{-1}{(1-x)\ln(1-x)}$$

29. **B** Note that if we plug in zero, we get a limit of the indeterminate form $\frac{0}{0}$, so we can use L'Hôpital's Rule to find the limit. Take the derivative of the numerator and the denominator: $\lim_{x\to 0}\frac{xe^x}{1-e^x} = \lim_{x\to 0}\frac{xe^x + e^x}{-e^x}$. Now, if we plug in 0 we get: $\frac{0+1}{-1} = -1$.

30. **A** We can do this integral with u-substitution.

Step 1: Let $u = \tan x$. Then $du = \sec^2 x\, dx$.

Step 2: Substituting, we get $\int \tan^6 x \sec^2 x\, dx = \int u^6\, du$.

Step 3: This is an easy integral: $\int u^6\, du = \frac{u^7}{7} + C$.

Step 4: Substituting back, we get $\frac{\tan^7 x}{7} + C$.

31. **C** These are a pair of basic trigonometric integrals. You should have memorized several trigonometric integrals, particularly $\int \sin x\, dx = -\cos x + C$ and $\int \cos x\, dx = \sin x + C$.

Step 1: $\int_0^{\frac{\pi}{4}} \sin x\, dx + \int_{-\frac{\pi}{4}}^{0} \cos x\, dx = -\cos x\Big|_0^{\frac{\pi}{4}} + \sin x\Big|_{-\frac{\pi}{4}}^{0}$

Step 2: Now we evaluate the limits of integration, and we're done.

$$-\cos x\Big|_0^{\frac{\pi}{4}} + \sin x\Big|_{-\frac{\pi}{4}}^{0} = \left[\left(-\cos\frac{\pi}{4}\right) - (-\cos(0))\right] + \left[(\sin(0)) - \left(\sin\left(-\frac{\pi}{4}\right)\right)\right] = -\frac{1}{\sqrt{2}} + 1 + 0 + \frac{1}{\sqrt{2}} = 1$$

32. **D** First, we will need to find when the particle is moving up (velocity is positive) or down (velocity is negative). We can find the velocity by taking the derivative of the position: $v(t) = y'(t) = 3y^2 - 8y$. Next, set it equal to zero: $3y^2 - 8y = 0$.

Factor: $y(3y - 8) = 0$, so the velocity is zero at $y = 0$ and $y = \frac{8}{3}$. We can determine whether the velocity is positive or negative by using text values and plugging them into the derivative and checking the sign: $v(1) = 3(1)^2 - 8(1) < 0$ (moving down) and $v(3) = 3(3)^2 - 8(3) > 0$ (moving up). If we want to find the distance traveled, we now integrate the velocity function on the intervals, making the integral negative for when the particle is moving down. We get: $-\int_0^{\frac{8}{3}} 3y^2 - 8y\,dy + \int_{\frac{8}{3}}^{4} 3y^2 - 8y\,dy$.

33. **C** We can use the Fundamental Theorem of Calculus for Integrals to find this. First, note that $\int_0^4 f(x)\,dx + \int_4^8 f(x)\,dx = \int_0^8 f(x)\,dx$. Next note that if $\int_8^0 f(x)\,dx = -23$, then if we switch the limits of integration $\int_0^8 f(x)\,dx = 23$. Now we plug in: $10 + \int_4^8 f(x)\,dx = 23$, so $\int_4^8 f(x)\,dx = 13$.

34. **B** First, we need to integrate: $f(x) = \int 3\sin x + 2\cos x\,dx = -3\cos x + 2\sin x + C$. Next, plug in the initial condition to solve for the constant.

$$5 = -3\cos(0) + 2\sin(0) + C$$

$$5 = -3(1) + C$$

$$C = 8$$

Therefore, $f(x) = -3\cos x + 2\sin x + 8$.

35. **A** The curve has a cusp at $x = 5$, so the curve is continuous there. But the slope is infinite at a cusp, so there is no derivative there. This means that $f'(5)$ is undefined.

36. **C** We need to integrate $f(x) = e^{-\frac{x^2}{4}}$ from $x = -1.65$ to $x = 1.65$. We can do this with the calculator. We get $\int_{-1.65}^{1.65} e^{-\frac{x^2}{4}}\,dx = 2.682$.

37. **A** This problem requires you to be familiar with the Mean Value Theorem for Integrals, which we use to find the average value of a function.

Step 1: If you want to find the average value of $f(x)$ on an interval $[a, b]$, you need to evaluate the integral $\frac{1}{b-a}\int_a^b f(x)\,dx$. So here we evaluate the integral $\frac{1}{2}\int_2^4 \ln^2 x\,dx$.

You have to do this integral on your calculator because you do not know how to evaluate this integral analytically unless you are very good with integration by parts!

Use **fnint.** Divide this by 2, and you will get 1.204.

38. **D** This problem is testing your knowledge of the Second Fundamental Theorem of Calculus. The theorem states that $\frac{d}{dx}\int_a^u f(t)\,dt = f(u)\frac{du}{dx}$, where a is a constant and u is a function of x. So all we have to do is follow the theorem: $\frac{d}{dx}\int_0^{3x} \cos(t)\,dt = 3\cos 3x$.

39. **A** If $f'(x)$ is positive, then the curve will have an upward (positive) slope. If $f'(x)$ is decreasing, it means that the second derivative is negative, so the curve will be concave down. Point a has both of those attributes.

40. **A** This is not a related-rate problem; this is a differential equation! It just happens to involve a rate.

Step 1: If we translate the first sentence into an equation, we get $\frac{dR}{dt} = kR$.

Put all of the terms that contain an R on the left of the equals sign and all of the terms that contain a t on the right-hand side.

$$\frac{dR}{R} = k\,dt$$

Step 2: Integrate both sides.

$$\int \frac{dR}{R} = k\int dt$$

Step 3: If we solve this for R, we get $R = Ce^{kt}$ (see the unit on Differential Equations).

Now we need to solve for C and k. First, we solve for C by plugging in the information that the radius is 4 initially. This means that $R = 4$ when $t = 0$.

If $4 = Ce^0$, then $C = 4$

Next, we solve for k by plugging in the information that $R = 10$ when $t = 2$.

$$10 = 4e^{2k}$$

$$\frac{5}{2} = e^{2k}$$

$$\ln\frac{5}{2} = 2k$$

$$\frac{1}{2}\ln\frac{5}{2} = k$$

Step 4: Now we have our final equation: $R = 4e^{\left(\frac{1}{2}\ln\frac{5}{2}\right)t}$.

If we plug in $t = 3$, we get $R = 4e^{\left(\frac{1}{2}\ln\frac{5}{2}\right)(3)} \approx 15.81$.

41. **A** We use the Second Fundamental Theorem of Calculus to find the derivative of the integral. In this case, we simply replace t in the integrand with x: $f'(x) = x^3 + x$. Now, all we have to do is plug in $x = 5$: $f'(5) = 5^3 + 5 = 130$.

42. **C** This is a simple integral that we do using integration by parts. You should memorize that $\int \ln(ax)\,dx = x\ln(ax) - x + C$, which makes this integral easy.

Step 1: The formula for integration by parts is $\int u\,dv = uv - \int v\,du$.

The trick is that we have to let $dv = dx$.

$$\text{Let } u = \ln 2x \text{ and } dv = dx$$

$$du = \frac{2}{2x}\,dx = \frac{1}{x}\,dx \text{ and } v = x$$

Plugging in to the formula we get

$$\int \ln 2x\,dx = x\ln 2x - \int dx = x\ln 2x - x + C$$

43. **B** First, in order to be differentiable, f has to be continuous. Here, this means that if we plug 2 into both pieces of f, we need to get the same value: $a(2)^2 + 3b(2) + 14 = 3a(2) + 5b$, which simplifies to

$$4a + 6b + 14 = 6a + 5b$$
$$2a - b = 14$$

Next, in order to be differentiable, if we plug 2 into both pieces of f', we need to get the same value. First, differentiate both pieces of f: $f'(x) = \begin{cases} 2ax + 3b; & x < 2 \\ 3a; & x > 2 \end{cases}$. Now we plug in 2 and set the two pieces equal to each other: $4a + 3b = 3a$, so $a = -3b$. Plug this into the first equation: $2(-3b) - b = 14$. Thus, $b = -2$ and $a = 6$.

44. **D** This is a Related Rates problem. We need to find out how fast the circumference of a circle, C, is increasing given how fast its area, A, is increasing. We are given that $\frac{dA}{dt} = 640$ and that $y = 100$. Now, we need to find the relationship between the area of a circle and its circumference. The area of a circle is $A = \pi r^2$ and the circumference is $C = 2\pi r$, where r is the radius.

Although we could isolate r in the second equation and substitute it into the first equation to get a direct relationship between A and C, it is easier to differentiate them separately and use $\frac{dr}{dt}$. That is, we first differentiate the area equation with respect to time: $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ and plug in. We get $640 = 2\pi(100)\frac{dr}{dt}$, so $\frac{dr}{dt} = \frac{640}{200\pi}$. Next, we differentiate the circumference equation with respect to time: $\frac{dC}{dt} = 2\pi \frac{dr}{dt}$. Now if we plug in $\frac{dr}{dt} = \frac{640}{200\pi}$, we get $\frac{dC}{dt} = 2\pi\left(\frac{640}{200\pi}\right) = 6.400$.

45. **B** **Step 1:** If we want to find the distance traveled, we take the integral of velocity from the starting time to the finishing time. Therefore, we need to evaluate $\int_0^4 7e^{-t^2}\,dt$.

Step 2: But we have a problem! We can't take the integral of e^{-t^2}. This means that the AP Exam wants you to find the answer using your calculator.

Rounded to three decimal places, the answer is 6.204.

ANSWERS AND EXPLANATIONS TO SECTION II

1. A particle moves along the x-axis so that its acceleration at any time $t > 0$ is given by $a(t) = 12t - 18$. At time $t = 1$, the velocity of the particle is $v(1) = 0$ and the position is $x(1) = 9$.

(a) Write an expression for the velocity of the particle $v(t)$.

Step 1: We know that the derivative of velocity with respect to time is acceleration, so the integral of acceleration with respect to time is velocity.

$$\int a(t)\,dt = v(t)$$
$$\int 12t - 18\,dt = 6t^2 - 18t + C = v(t)$$

If we plug in the information that at time $t = 1$, $v(1) = 0$, we can solve for C.

$$6(1)^2 - 18(1) + C = 0$$
$$-12 + C = 0$$
$$C = 12$$

This means that the velocity of the particle is $6t^2 - 18t + 12$.

(b) At what values of t does the particle change direction?

When a particle is in motion, it changes direction at the time when its velocity is zero. (As long as acceleration is not also zero.) So all we have to do is set velocity equal to zero and solve for t.

$$\begin{aligned} 6t^2 - 18t + 12 &= 0 \\ t^2 - 3t + 2 &= 0 \\ (t-2)(t-1) &= 0 \\ t &= 1, 2 \end{aligned}$$

(c) Write an expression for the position $x(t)$ of the particle.

We know that the derivative of position with respect to time is velocity, so the integral of velocity with respect to time is position.

$$\int v(t)\,dt = x(t)$$

$$\int (6t^2 - 18t + 12)\,dt = 2t^3 - 9t^2 + 12t + C = x(t)$$

If we plug in the information that at time $t = 1$, $x(1) = 9$, we can solve for C.

$$\begin{aligned} 2(1)^3 - 9(1)^2 + 12(1) + C &= 9 \\ 5 + C &= 9 \\ C &= 4 \end{aligned}$$

Therefore, $x(t) = 2t^3 - 9t^2 + 12t + 4$.

(d) Find the total distance traveled by the particle from $t = \frac{3}{2}$ to $t = 6$.

Step 1: Normally, all that we have to do to find the distance traveled is to integrate the velocity equation from the starting time to the ending time. But we have to watch out for whether the particle changes direction. If so, we have to break the integration into two parts—a positive integral for when it is traveling to the right, and a negative integral for when it is traveling to the left.

One way to solve this is to find two integrals and add them together. Because you can use a calculator, it is simpler to use the fnInt calculation of the absolute value for $t = \frac{3}{2}$ and $t = 6$, using the function of velocity $6x^2 - 18x + 12$.

Graphing Calculator (TI-83 and TI-84)

Press MATH and select 9: fnInt from the list.

Press MATH then select the NUM menu, and choose 1: abs(

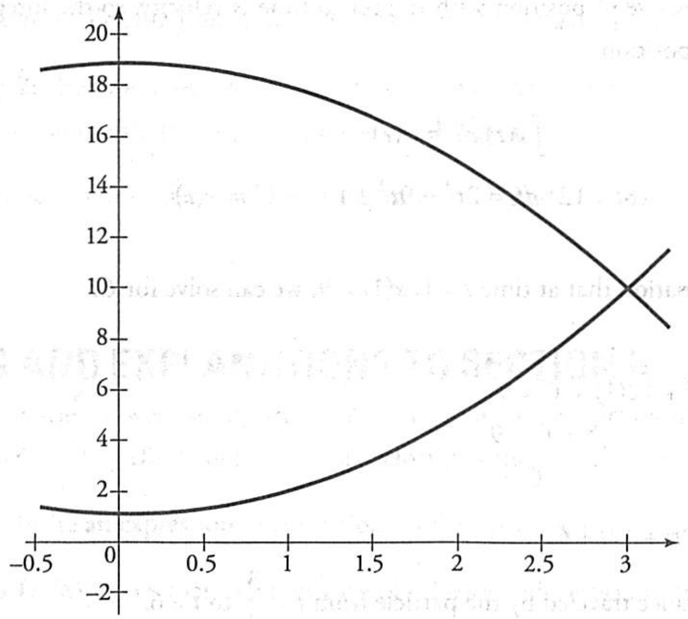
Enter the function $6x^2 - 18x + 12$ and follow with the closing parentheses. List the variable and low and high values for t, separated by commas, and follow with final closing parentheses so your expression looks like the following:

fnInt(abs($6x^2 - 18x + 12$), x, 3/2, 6)

Press ENTER

The result is 176.500.

2. Let R be the region in the first quadrant bounded from above by $g(x) = 19 - x^2$ and from below by $f(x) = x^2 + 1$.



(a) Find the area of R.

First, let's find where the curves intersect. Set them equal to each other:

$$19 - x^2 = x^2 + 1$$

$$2x^2 = 18$$

$$x^2 = 9$$

$$x = \pm 3$$

We are only interested in the first quadrant, so in order to find the area, we need to evaluate the integral $\int_0^3 (19 - x^2) - (x^2 + 1)dx$. We get:

$$\int_0^3 (19 - x^2) - (x^2 + 1)dx = \int_0^3 (18 - 2x^2)dx = \left(18x - \frac{2x^3}{3}\right)\Bigg|_0^3 = (54 - 18) - (0 - 0) = 36$$

(b) A solid is formed by revolving R around the x-axis. Find the volume of the solid.

In order to find the volume, we need to evaluate the integral $\pi\int_0^3 (19 - x^2)^2 - (x^2 + 1)^2 dx$. We get:

$$\pi\int_0^3 (361 - 38x^2 + x^4) - (x^4 + 2x^2 + 1)dx =$$

$$\pi\int_0^3 (360 - 40x^2)dx =$$

$$\pi\int_0^3 (360 - 40x^2)dx = \pi\left(360x - \frac{40x^3}{3}\right)\Bigg|_0^3 = \pi(1080 - 360) - \pi(0 - 0) = 720\pi$$

(c) A solid has its base as the region R, whose cross-sections perpendicular to the x-axis are squares. Find the volume of the solid.

In order to find the volume, we need to find the area of each cross-section, which is *side*^{2} (because it is a square), where the side is the difference between the upper and lower curve. Thus, we need to evaluate the integral $\int_0^3 \left[(19 - x^2) - (x^2 + 1)\right]^2 dx$. We get:

$$\int_0^3 \left[(19 - x^2) - (x^2 + 1)\right]^2 dx =$$

$$\int_0^3 (18 - 2x^2)^2 dx =$$

$$\int_0^3 (324 - 72x^2 + 4x^4)dx =$$

$$\left(324x - 24x^3 + \frac{4x^5}{5}\right)\Bigg|_0^3 = \left(972 - 648 + \frac{324}{5}\right) - (0 - 0) = 324 + \frac{324}{5} = \frac{1944}{5}$$

3. Consider the equation $x^2 - 2xy + 4y^2 = 84$.

(a) Write an expression for the slope of the curve at any point (x, y).

Step 1: The slope of the curve is just the derivative. But here we have to use implicit differentiation to find the derivative. If we take the derivative of each term with respect to x, we get

$$2x\frac{dx}{dx} - 2\left(x\frac{dy}{dx} + y\frac{dx}{dx}\right) + 8y\frac{dy}{dx} = 0$$

Remember that $\frac{dx}{dx} = 1$, which gives us

$$2x - 2\left(x\frac{dy}{dx} + y\right) + 8y\frac{dy}{dx} = 0$$

Step 2: Now just simplify and solve for $\frac{dy}{dx}$.

$$2x - 2x\frac{dy}{dx} - 2y + 8y\frac{dy}{dx} = 0$$

$$x - x\frac{dy}{dx} - y + 4y\frac{dy}{dx} = 0$$

$$-x\frac{dy}{dx} + 4y\frac{dy}{dx} = y - x$$

$$(4y - x)\frac{dy}{dx} = y - x$$

$$\frac{dy}{dx} = \frac{y - x}{4y - x}$$

(b) Find the equation of the tangent lines to the curve at the point $x = 2$.

Step 1: We are going to use the point-slope form of a line, $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a point on the curve, and the derivative at that point is the slope m. First, we need to know the value of y when $x = 2$. If we plug 2 for x into the original equation, we get

$$4 - 4y + 4y^2 = 84$$

$$4y^2 - 4y - 80 = 0$$

$$y^2 - y - 20 = 0$$

This factors to $(y - 5)(y + 4) = 0$, so $y = 5$ or $y = -4$.

Notice that there are two values of y when $x = 2$, which is why there are two tangent lines.

Step 2: Now that we have our points, we need the slope of the tangent line at $x = 2$.

$$\frac{dy}{dx} = \frac{y - x}{4y - x}$$

At (2, 5), $\frac{dy}{dx} = \frac{5 - 2}{4(5) - 2} = \frac{3}{18} = \frac{1}{6}$

At (2, –4), $\frac{dy}{dx} = \frac{-4 - 2}{4(-4) - 2} = \frac{-6}{-18} = \frac{1}{3}$

Step 3: Plugging into our equation for the tangent line, we get

$$y - 5 = \frac{1}{6}(x - 2)$$

$$y + 4 = \frac{1}{3}(x - 2)$$

It is not necessary to simplify these equations.

(c) Find $\frac{d^2y}{dx^2}$ at (0, 4).

Step 1: Once we have the first derivative, we have to differentiate again to find $\frac{d^2y}{dx^2}$.

We have to use implicit differentiation again.

$$\frac{dy}{dx} = \frac{y - x}{4y - x}$$

Use the Quotient Rule.

$$\frac{d^2y}{dx^2} = \frac{(4y - x)\left(\frac{dy}{dx} - \frac{dx}{dx}\right) - (y - x)\left(4\frac{dy}{dx} - \frac{dx}{dx}\right)}{(4y - x)^2}$$

Simplifying, we get

$$\frac{d^2y}{dx^2} = \frac{(4y - x)\left(\frac{dy}{dx} - 1\right) - (y - x)\left(4\frac{dy}{dx} - 1\right)}{(4y - x)^2}$$

Now we plug in $\frac{y - x}{4y - x}$ for $\frac{dy}{dx}$, which gives us

$$\frac{d^2y}{dx^2} = \frac{(4y - x)\left(\frac{y - x}{4y - x} - 1\right) - (y - x)\left(4\frac{y - x}{4y - x} - 1\right)}{(4y - x)^2}$$

Now we would have to use a lot of algebra to simplify this but, fortunately, we can just plug in $(0, \sqrt{21})$ for x and y and solve from there.

$$\frac{d^2y}{dx^2} = \frac{(4\sqrt{21})\left(\frac{\sqrt{21}}{4\sqrt{21}} - 1\right) - (\sqrt{21})\left(4\frac{\sqrt{21}}{4\sqrt{21}} - 1\right)}{(4\sqrt{21})^2} = \frac{-3\sqrt{21}}{336}$$

4. Water is draining at the rate of 48π ft^3/sec from the vertex at the bottom of a conical tank whose diameter at its base is 40 feet and whose height is 60 feet.

(a) Find an expression for the volume of water (in ft^3/sec) in the tank, in terms of its radius, at the surface of the water.

The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 H$, where r is the radius of the cone and h is the height. The ratio of the height of a cone to its radius is constant at any point on the edge of the cone, so we also know that $\frac{h}{r} = \frac{60}{20} = 3$. (Remember that the radius is half the diameter.) If we solve this for h and substitute, we get

$$h = 3r$$

$$V = \frac{1}{3}\pi r^2 (3r) = \pi r^3$$

(b) At what rate (in ft/sec) is the radius of the water in the tank shrinking when the radius is 16 feet?

Step 1: This is a related rates question. We now have a formula for the volume of the cone in terms of its radius, so if we differentiate it in terms of t, we should be able to solve for the rate of change of the radius $\frac{dr}{dt}$.

We are given that the rate of change of the volume and the radius are, respectively

$$\frac{dV}{dt} = -48\pi \text{ and } r = 16$$

Differentiating the formula for the volume, we get $\frac{dV}{dt} = 3\pi r^2 \frac{dr}{dt}$.

Now, we plug in and get $-48\pi = 3\pi(16)^2 \frac{dr}{dt}$. Finally, if we solve for $\frac{dr}{dt}$, we get

$$\frac{dr}{dt} = -\frac{1}{16} \text{ ft/sec}$$

(c) How fast (in ft/sec) is the height of the water in the tank dropping at the instant that the radius is 16 feet?

Step 1: This is the same idea as the previous problem, except that we want to solve for $\frac{dh}{dt}$.

In order to do this, we need to go back to our ratio of height to radius and solve it for the radius.

$$\frac{h}{r} = 3 \quad \text{or} \quad \frac{h}{3} = r$$

Substituting for r in the original equation, we get $V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h = \frac{\pi h^3}{27}$.

Step 2: Now we need to know what h is when r is 16. Using our ratio,

$$h = 3(16) = 48.$$

Step 3: Now if we differentiate, we get

$$\frac{dV}{dt} = \frac{\pi h^2}{9}\frac{dh}{dt}$$

Now we plug in and solve.

$$48\pi = \frac{\pi(48)^2}{9}\frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{3}{16}$$

One should also note that, because $h = 3r$, $\frac{dh}{dt} = 3\frac{dr}{dt}$. Thus, after we found $\frac{dr}{dt}$ in part 2, we merely had to multiply it by 3 to find the answer for part 3.

5. Let f be the function given by $y = f(x) = 2x^4 - 4x^2 + 1$.

(a) Find an equation of the line tangent to the graph at $(-2, 17)$.

In order to find the equation of a tangent line at a particular point, we need to take the derivative of the function and plug in the x- and y-values at that point to give us the slope of the line.

Step 1: The derivative is $f'(x) = 8x^3 - 8x$. If we plug in $x = -2$, we get

$$f'(-2) = 8(-2)^3 - 8(-2) = -48$$

This is the slope m.

Step 2: Now we use the slope-intercept form of the equation of a line, $y - y_1 = m(x - x_1)$, and plug in the appropriate values of x, y, and m.

$$y - 17 = -48(x + 2)$$

If we simplify this, we get $y = -48x - 79$.

(b) Find the x- and y-coordinates of the relative maxima and relative minima. Verify your answer.

If we want to find the maxima/minima, we need to take the derivative and set it equal to zero. The values that we get are called critical points. We will then test each point to see if it is a maximum or a minimum.

Step 1: We already have the first derivative from part (a), so we can just set it equal to zero.

$$8x^3 - 8x = 0$$

If we now solve this for x, we get

$$8x(x^2 - 1) = 0$$
$$8x(x + 1)(x - 1) = 0$$
$$x = 0,\ 1,\ -1$$

These are our critical points. In order to test if a point is a maximum or a minimum, we usually use the *second derivative test.* We plug each of the critical points into the second derivative. If we get a positive value, the point is a relative minimum. If we get a negative value, the point is a relative maximum. If we get zero, the point is a point of inflection.

Step 2: The second derivative is $f''(x) = 24x^2 - 8$. If we plug in the critical points, we get

$$f''(0) = 24(0)^2 - 8 = -8$$

$$f''(1) = 24(1)^2 - 8 = 16$$

$$f''(-1) = 24(-1)^2 - 8 = 16$$

So $x = 0$ is a relative maximum, and $x = 1, -1$ are relative minima.

Step 3: In order to find the y-coordinates, we plug the x-values back into the original equation and solve.

$$f(0) = 1$$
$$f(1) = -1$$
$$f(-1) = -1$$

And our points are

(0, 1) is a relative maximum

(1, –1) is a relative minimum

(–1, –1) is a relative minimum

(c) Find the x- and y- coordinates of the points of inflection. Verify your answer.

If we want to find the points of inflection, set the second derivative equal to zero. The values that we get are the x-coordinates of the points of inflection.

Step 1: We already have the second derivative from part (b), so all we have to do is set it equal to zero and solve for x.

$$24x^2 - 8 = 0$$

$$x^2 = \frac{1}{3}$$

$$x = \pm\sqrt{\frac{1}{3}}$$

Step 2: In order to find the y-coordinates, plug the x-values back into the original equation and solve.

$$f\left(\sqrt{\frac{1}{3}}\right) = 2\left(\sqrt{\frac{1}{3}}\right)^4 - 4\left(\sqrt{\frac{1}{3}}\right)^2 + 1 = \frac{2}{9} - \frac{4}{3} + 1 = -\frac{1}{9}$$

$$f\left(-\sqrt{\frac{1}{3}}\right) = 2\left(-\sqrt{\frac{1}{3}}\right)^4 - 4\left(-\sqrt{\frac{1}{3}}\right)^2 + 1 = \frac{2}{9} - \frac{4}{3} + 1 = -\frac{1}{9}$$

So the points of inflection are $\left(\sqrt{\frac{1}{3}}, -\frac{1}{9}\right)$ and $\left(-\sqrt{\frac{1}{3}}, -\frac{1}{9}\right)$.

6. Grain is being loaded into a silo at the rate of $G(t) = 400e^{\frac{-t^2}{4}}\ ft^3 / hr$, where t is the number of hours that it is being loaded, $0 \le t \le 8$. At time $t = 0$, there is 100 ft^3 of grain in the silo. Grain is also being removed through the base of the silo at the following rates, where $R(t)$ is the amount of grain being removed in ft^3/hr, $0 \le t \le 8$:

t	0	2	5	7	8
$R(t)$	60	90	110	120	125

(a) Estimate the total amount of grain removed from the silo at $t = 8$ hrs, using a left-hand Riemann Sum and 4 subintervals.

The Riemann Sum is found by multiplying the width of each interval by R at each left endpoint of an interval. We get:

$(2 - 0)(60) + (5 - 2)(90) + (7 - 5)(110) + (8 - 7)(120)$

$(2)(60) + (3)(90) + (2)(110) + (1)(120) = 120 + 270 + 220 + 120 = 730.$

(b) Estimate the amount of grain in the silo at the end of 8 hours, using your answer from part (a).

We can estimate the amount of grain in the silo by taking the starting amount, 100, adding the amount that comes in, and subtracting the amount that is being removed (which we found in part a). We can find the amount that comes in by $\int_0^8 400e^{\frac{-t^2}{4}}\,dt$. If you plug this in your calculator, you get 708.982. (If you are unsure how to evaluate the integral on your calculator, check the Appendix for some tips on using a TI-84 calculator.) Therefore, the amount in the silo at the end of 8 hours is $100 + 708.982 - 730 = 78.982$.

(c) Estimate $R'(5)$, showing your work. Indicate the units of measure.

We can estimate $R'(5)$ by finding the slope of the secant line from $t = 2$ to $t = 5$, and from $t = 5$ to $t = 7$ and averaging the two. The slope of the secant line from $t = 2$ to $t = 5$ is $\frac{110-90}{5-2} = \frac{20}{3}$, and the slope of the secant line from $t = 5$ to $t = 7$ is $\frac{120-110}{7-5} = 5$, so a good estimate for $R'(5)$ is $\frac{5+\frac{20}{3}}{2} = \frac{35}{6}$.