6.5 Arithmetic Sequence

REVIEW Math 10

A **sequence** is simply a list of numbers. In a sequence each number is called a **term** of the sequence. There is a first term, second term, third term, and so on. A sequence can be **finite**, in which it is possible to count the number of terms, or **infinite**, in which the terms continue forever.

For example: 1, 3, 6, 10 is a finite sequence
1, 3, 6, 10, ... is an infinite sequence

A sequence is a function whose domain is a set of positive integers. However, a sequence is written using subscript notation rather than function notation.

For example: $a_1, a_2, a_3, ..., a_n$

The subscript identifies the term of the sequence. For instance a_3 is the third term, and a_n is the nth term of the sequence. The entire sequence is usually denoted by $\{a_n\}$.

Sequence

A **finite sequence** is a function for which the domain is the subset of natural numbers: $\{1, 2, 3, ..., n\}$ for some finite number n.

An **infinite sequence** is a function for which the domain is the set of natural numbers: $\{1, 2, 3, ...\}$.

Example 1 Write the first four terms of the sequence.

a) $a_n = \dfrac{n+1}{n}$

b) $b_n = 2n - 3$

c) $t_n = 2^n$

▶ *Solution:* **a)** $a_1 = \dfrac{1+1}{1} = 2,\ a_2 = \dfrac{2+1}{2} = \dfrac{3}{2},\ a_3 = \dfrac{3+1}{3} = \dfrac{4}{3},\ a_4 = \dfrac{4+1}{4} = \dfrac{5}{4}$

b) $b_1 = 2(1) - 3 = -1,\ b_2 = 2(2) - 3 = 1,\ b_3 = 2(3) - 3 = 3,\ b_4 = 2(4) - 3 = 5$

c) $t_1 = 2^1 = 2,\ t_2 = 2^2 = 4,\ t_3 = 2^3 = 8,\ t_4 = 2^4 = 16$

Another way of defining a sequence is to define the first term, or the first few terms, and specify the nth term by a formula involving the preceding term(s). Sequences defined in this manner are called **recursive**.

Example 2 Write the first four terms of the recursive formula: $a_1 = 3,\ a_n = \dfrac{a_{n-1}}{n}$.

▶ *Solution:* $a_1 = 3,\ a_2 = \dfrac{a_{2-1}}{2} = \dfrac{a_1}{2} = \dfrac{3}{2},\ a_3 = \dfrac{a_{3-1}}{3} = \dfrac{a_2}{3} = \dfrac{1}{2},\ a_4 = \dfrac{a_{4-1}}{4} = \dfrac{a_3}{4} = \dfrac{1}{8}$

Sigma Notation

It is often important to find the sum of a sequence, $\{t_n\} = a_1 + a_2 + a_3 + \cdots + a_n$. The expanded notation $a_1 + a_2 + a_3 + \cdots + a_n$ can be written more compactly using **sigma notation**. The Greek letter Σ (sigma) is used as the summation symbol in sigma notation.

$$\underbrace{a_1 + a_2 + a_3 + \cdots + a_n}_{\textit{expanded notation}} = \underbrace{\sum_{k=1}^{n} a_k}_{\textit{sigma notation}}$$

The integer k is called the index of the sum, which shows where the summation starts. The integer n shows where the summation ends.

The summation $\sum_{k=1}^{n} a_k$ has $n - k + 1$ terms.

Example 3 Find the sum of each sequence.

a) $\sum_{k=1}^{4}(2k+1)$

b) $\sum_{k=1}^{5}(k^2+1)$

c) $\sum_{k=1}^{3}(k^3-k)$

▶ *Solution:*

a) $2(1)+1=3,\quad 2(2)+1=5,\quad 2(3)+1=7,\quad 2(4)+1=9$

$3+5+7+9=24$

b) $1^2+1=2,\quad 2^2+1=5,\quad 3^2+1=10,\quad 4^2+1=17,\quad 5^2+1=26$

$2+5+10+17+26=60$

c) $1^3-1=0,\quad 2^3-2=6,\quad 3^3-3=24$

$0+6+24=30$

Example 4 Write the sum using sigma notation.

a) $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots+\frac{12}{12+1}$

b) $\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\cdots+\left(\frac{2}{3}\right)^n$

▶ *Solution:*

a) $\sum_{k=1}^{12}\frac{k}{k+1}$

b) $\sum_{k=1}^{n}\left(\frac{2}{3}\right)^k$

Arithmetic Sequence

When the difference between successive terms of a sequence is always the same number, the sequence is called **arithmetic**. For example the sequence 3, 7, 11, 15, ... is arithmetic because adding 4 to any term produces the next term. The **common difference**, d, of this sequence is 4.

To develop a formula to find the general term of an arithmetic sequence, the first few terms need to be expanded.

1st term: $a_1 = a_1$
2nd term: $a_2 = a_1 + d$
3rd term: $a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d$
4th term: $a_4 = a_3 + d = (a_1 + 2d) + d = a_1 + 3d$

Notice that the coefficient of d is one less than the subscript of the term.

The nth Term of an Arithmetic Sequence

For an arithmetic sequence $\{t_n\}$ whose first term is a, with common difference d:

$$t_n = a + (n-1)d \quad \text{for any integer } n \geq 1$$

Example 5 For each arithmetic sequence, identify the common difference.

a) 3, 5, 7, 9, ...

b) 11, 8, 5, 2, ...

▶ *Solution*: **a)** $5-3=2, \quad 7-5=2, \quad 9-7=2$

Therefore $d = 2$.

b) $8-11=-3, \quad 5-8=-3, \quad 2-5=-3$

Therefore $d=-3$.

Example 6 Determine if the sequence $\{t_n\} = \{3-2n\}$ is arithmetic.

▶ *Solution*: $t_1 = a = 3-2(1) = 1$

$t_2 = a_2 = 3-2(2) = -1$

$t_3 = a_3 = 3-2(3) = -3$

$1, -1, -3, \ldots$ has a common difference of -2, therefore the sequence is arithmetic.

Example 7 Find the 12th term of the arithmetic sequence 2, 5, 8, ...

▶ *Solution*: $a_1 = a = 2$

$d = a_2 - a_1 = 5 - 2 = 3$

$t_n = a + (n-1)d$

$t_{12} = 2 + (12-1)\cdot 3$

$= 35$

The 12th term is 35.

Example 8 Which term in the arithmetic sequence 4, 7, 10, ... has a value of 439?

▶ *Solution*: $d = 7 - 4 = 3$

$t_n = a + (n-1)d$

$439 = 4 + (n-1)\cdot 3$

$435 = 3(n-1)$

$145 = n - 1$

$n = 146$

The 146th term is 439.

Example 9 The 7th term of an arithmetic sequence is 78, and the 18th term is 45. Find the first term.

▶ *Solution*: $t_n = a + (n-1)d$

$t_7 = a + 6d = 78$

$t_{18} = \underline{a + 17d = 45}$

$-11d = 33$

$d = -3$

$t_7 = a + 6d = 78$

$= a + 6(-3) = 78$

$a = 78 + 18$

$= 96$

The first term is 96.

Example 10 Find x so that $3x + 2$, $2x - 3$, and $2 - 4x$ are consecutive terms of an arithmetic sequence.

▶ *Solution*: If a, b, and c are three consecutive terms of an arithmetic sequence, then $\frac{a+c}{2} = b$.

$\frac{(3x+2)+(2-4x)}{2} = 2x - 3$

$-x + 4 = 4x - 6$

$-5x = -10$

$x = 2$

Check: $3(2) + 2 = 8$, $2(2) - 3 = 1$, and $2 - 4(2) = -6$

8, 1, -6 is an arithmetic sequence with $d = -7$

6.5 Exercise Set

1. Fill in the blanks.

   a) The domain of a sequence is the set of consecutive ____________ numbers.

   b) A sequence with a last term is a(n) ____________ sequence.

   c) A sequence with no last term is a(n) ____________ sequence.

   d) The sequence $a_1 = 2,\ a_n = 2a_{n-1}$ is a ____________ sequence.

   e) The formula for the nth term of an arithmetic sequence is $t_n =$ ______________ .

2. Write the first four terms of each sequence.

   a) $\{n^2 - 2\}$

   b) $\left\{\frac{n+2}{n+1}\right\}$

   c) $\{(-1)^{n+1}n^2\}$

   d) $\left\{\frac{3^n}{2^n+1}\right\}$

   e) $\left\{\frac{2^n}{n^2}\right\}$

   f) $\left\{\left(\frac{2}{3}\right)^n\right\}$

3. Write the nth term of the suggested pattern.

   a) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

   b) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

   c) $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \ldots$

   d) $2, -4, 6, -8, \ldots$

4. Write the first four terms of the recursive sequence.

   a) $a = 4,\ t_n = 2 + t_{n-1}$

   b) $a = 3,\ t_n = n - t_{n-1}$

   c) $a = 2,\ a_2 = 3,\ a_n = a_{n-1} + a_{n-2}$

   d) $a_1 = -1,\ a_2 = 1,\ a_n = na_{n-1} + a_{n-2}$

5. Find the sum of each sequence.

a) $\sum_{k=1}^{5} 4$

b) $\sum_{k=1}^{4} (k^2 - 2)$

c) $\sum_{k=2}^{5} (k^2 - 1)$

d) $\sum_{k=0}^{3} (k^3 - 1)$

e) $\sum_{k=1}^{4} \frac{k^2}{2}$

f) $\sum_{k=6}^{8} (k+1)^2$

6. Express each sum using summation notation with index $k = 1$.

a) $1 + 3 + 5 + 7$

b) $1^2 + 2^2 + 3^2 + 4^2 + 5^2$

c) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{n}{n+1}$

d) $5 + \frac{5^2}{2} + \frac{5^3}{3} + \cdots + \frac{5^n}{n}$

7. Write the first five terms of each arithmetic sequence.

a) $7, 11, 15,$ ____, ____

b) $15, 12, 9,$ ____, ____

c) $a = 4,\ d = 2$

d) $a = -1,\ d = -3$

e) $a = -5,\ d = -\frac{3}{4}$

f) $a = -\frac{2}{3},\ d = \frac{1}{5}$

8. Find the indicated arithmetic term.

a) $a = 5,\ d = 3$; find t_{12}

b) $a = \frac{2}{3},\ d = -\frac{1}{4}$; find t_9

c) $a = -\frac{3}{4},\ d = \frac{1}{2}$; find t_{10}

d) $a = 2.5,\ d = -1.25$; find t_{20}

e) $a = -0.75,\ d = 0.05$; find t_{40}

f) $a = -1\frac{3}{4},\ d = -\frac{2}{3}$; find t_{37}

9. Find the number of terms in each arithmetic sequence.

a) $a = 6,\ t_n = -30,\ d = -3$

b) $a = -3,\ t_n = 82,\ d = 5$

c) $a = 0.6,\ t_n = 9.2,\ d = 0.2$

d) $a = -0.3,\ t_n = -39.4,\ d = -2.3$

e) $-1, 4, 9, \ldots, 159$

f) $23, 20, 17, \ldots, -100$

10. Find the first term in the arithmetic sequence.

a) 6th term is 10; 18th term is 46

b) 4th term is 2; 18th term is 30

c) 9th term is 23; 17th term is -1

d) 5th term is 3; 25th term is -57

e) 13th term is -3; 20th term is -17

f) 11th term is 37; 26th term is 32

11. Find x so that the values given are consecutive terms of an arithmetic sequence.

a) $x + 3,\ 2x + 1$, and $5x + 2$

b) $2x,\ 3x + 2$, and $5x + 3$

c) $x - 1,\ \frac{1}{2}x + 4$, and $1 - 2x$

d) $2x - 1,\ x + 1$, and $3x + 9$

e) $x + 4,\ x^2 + 5$, and $x + 30$

f) $8x + 7,\ 2x + 5$, and $2x^2 + x$

12. If t_n is a term of an arithmetic sequence, what is $t_n - t_{n-1}$ equal to?

13. List the first seven numbers of the Fibonacci sequence $a_1 = 1,\ a_2 = 1,\ a_n = a_{n-1} + a_{n-2},\ n > 2$.

14. The starting salary of an employee is \$23 750. If each year a \$1250 raise is given, in how many years will the employee's salary be \$50 000?

15. An auditorium has 8 seats in the first row. Each subsequent row has 4 more seats than the previous row. What row has 140 seats?

16. A well drilling company charges \$8.00 for the first meter, then \$8.75 for the second meter, and so on in an arithmetic sequence. At this rate, what would be the cost to drill the last meter of a well 120 meters deep?

17. It is said that during the last weeks of his life Abraham deMoivre needed 15 minutes more sleep each night, and when he needed 24 hours sleep he would die. If he needed 8 hours sleep on September 1, what day did he die?

18. The first three terms of an arithmetic sequence are $x - 3$, $\frac{x^2}{25} + 9$, and $3x - 11$. Determine the fourth term.

19. The first, third, and fifth terms of an arithmetic sequence are $2x - 1$, $x^2 - 3$, and $11 - x^2$ respectively. Determine the second term.

6.6 Arithmetic Series

The indicated sum of the terms of a sequence is called a **series**. For example $3 + 7 + 11 + 15$ is a series. Just as a sequence may be finite or infinite, a series can also be finite or infinite. However, this section will only discuss finite series.

Deriving the Sum Formula for Finite Arithmetic Series

If $a_1, a_2, a_3, \ldots, a_n$ is a finite arithmetic sequence, then $a_1 + a_2 + a_3 + \cdots + a_n$ is a finite arithmetic series.

Let $d =$ the common difference, $S_n =$ the sum of the series.

$$\begin{aligned} S_n &= a_1 + a_2 + a_3 + \cdots + a_n \\ &= a + (a+d) + (a+2d) + (a+3d) + \cdots + (a+(n-1)d) \quad \text{(equation 1)} \end{aligned}$$

Let $l = a + (n-1)d$ (*the last term*)

Writing the sum in reverse order: $S_n = l + (l-d) + (l-2d) + \cdots + a$ (equation 2)

Adding equations 1 and 2: $2S_n = (a+l) + (a+d+l-d) + (a+2d+l-2d) + \cdots + (a+l)$
$= (a+l) + (a+l) + (a+l) + \cdots + (a+l)$

But $(a+l)$ appears n times. Therefore $2S_n = n(a+l) \rightarrow S_n = \frac{n}{2}(a+l)$.

Also, $S_n = \frac{n}{2}(a+l) \rightarrow S_n = \frac{n}{2}(a + a + (n-1)d) \rightarrow S_n = \frac{n}{2}(2a + (n-1)d)$.

> **Sum of an Arithmetic Series**
>
> The sum of the first n terms of an arithmetic series is given by:
>
> $$S_n = \frac{n}{2}(a+l) = \frac{n}{2}(2a + (n-1)d)$$
>
> where $a =$ the first term, $l =$ the last term, and $d =$ the common difference

Example 1 Find the sum of the positive integers from 1 to 50 inclusive.

▶ *Solution*: $a = 1$, $d = 1$, and $l = 50$

$$\begin{aligned} S_n &= \frac{n}{2}(a+l) \\ S_{50} &= \frac{50}{2}(1+50) \\ &= 1275 \end{aligned}$$

Example 2 Find the sum of the first 25 terms of the series $11 + 15 + 19 + \cdots$

▶ *Solution*: This series is arithmetic with $a = 11$, $d = 4$, and $n = 25$

$$\begin{aligned} S_n &= \frac{n}{2}(2a + (n-1)d) \\ S_{25} &= \frac{25}{2}(2 \cdot 11 + (25-1)(4)) \\ &= \frac{25}{2}(22 + 96) \\ &= 1475 \end{aligned}$$

Example 3 Find the sum of the series $7 + 10 + 13 + \cdots + 100$.

▶ *Solution*: $a = 7$, $d = 3$, and $l = 100$

To find n:

$$\begin{aligned} l &= a + (n-1)d \\ 100 &= 7 + (n-1)(3) \\ 93 &= (n-1)(3) \\ 31 &= n - 1 \\ n &= 32 \end{aligned}$$

$$\begin{aligned} S_n &= \frac{n}{2}(a + l) \\ S_{32} &= \frac{32}{2}(7 + 100) \\ &= 1712 \end{aligned}$$

Summation

If the summation expression is a linear function, then the summation is an arithmetic series.

For example: $\sum_{k=1}^{10}(2k+1)$ is an arithmetic series since $f(x) = 2x + 1$ is linear.

$\sum_{k=1}^{10}(k^2+1)$ is not an arithmetic series since $f(x) = x^2 + 1$ is not linear.

Example 4 Evaluate $\sum_{k=1}^{100}(2k+1)$.

▶ *Solution*: $a = 2(1) + 1 = 3$, $l = 2(100) + 1 = 201$, $d = 2$, $n = 100$

$$\begin{aligned} S_n &= \frac{n}{2}(a + l) \\ S_{100} &= \frac{100}{2}(3 + 201) \\ &= 10\,200 \end{aligned}$$

Example 5 Write $5 + 9 + 13 + \cdots + 137$ in summation notation.

▶ *Solution*:

$$\begin{aligned} l &= a + (n-1)d \\ 137 &= 5 + (n-1)(4) \\ 132 &= (n-1)(4) \\ 33 &= n-1 \\ n &= 34 \end{aligned}$$

The linear expression must be in the form $ax + b$, with a common difference of $d = 4$. Therefore the linear expression is $4k + b$.

When $k = 1,\ 4(1) + b = 5 \rightarrow b = 1$

Then the expression is $4k + 1$, and the summation is $\sum_{k=1}^{34}(4k+1)$.

Example 6 The sum of the first n terms of an arithmetic sequence is $S_n = 5n^2 - 3n$. Find the common difference d.

▶ *Solution*:

$$\begin{aligned} S_n &= 5n^2 - 3n \\ S_1 &= a_1 = 5(1)^2 - 3(1) = 2 \\ S_2 &= 5(2)^2 - 3(2) = 14 \\ S_2 &= a_1 + a_2 = 14 \\ 2 + a_2 &= 14 \\ a_2 &= 12 \end{aligned}$$

If $a_1 = 2$, $a_2 = 12$, then $d = 12 - 2 = 10$

Example 7 Find the sum of all multiples of 6 between 100 and 1000.

▶ *Solution*: The first multiple of 6 is 102. The last multiple of 6 is 996.

$$\begin{aligned} t_n &= a + (n-1)d \\ 996 &= 102 + (n-1)\cdot 6 \\ n &= 150 \end{aligned}$$

$$\begin{aligned} S_{150} &= \frac{150}{2}(102 + 996) \\ &= 82\,350 \end{aligned}$$

Example 8 Find two arithmetic means between 8 and 29.

▶ *Solution*: $a_1 = 8,\quad a_2 = 8 + d,\quad a_3 = 8 + 2d,\quad a_4 = 8 + 3d = 29$

$8 + 3d = 29 \rightarrow d = 7$

Therefore the two arithmetic means are 15 and 22.

6.6 Exercise Set

1. Find the sum of the arithmetic series.

a) $3 + 5 + 7 + \cdots + (2n + 1)$

b) $-1 + 2 + 5 + \cdots + (3n - 4)$

c) $2 + 5 + 8 + \cdots + 77$

d) $5 + 9 + 13 + \cdots + 97$

e) $(-41) + (-35) + (-29) + \cdots + 541$

f) $2\sqrt{5} + 6\sqrt{5} + 10\sqrt{5} + \cdots + 50\sqrt{5}$

g) $39 + 33 + 27 + \cdots + (-15)$

h) $23 + 19 + 15 + \cdots + (-305)$

i) $\frac{1}{2} + \frac{7}{8} + \frac{5}{4} + \cdots + \frac{55}{8}$

j) $\frac{16}{3} + \frac{13}{3} + \frac{10}{3} + \cdots + \left(-\frac{65}{3}\right)$

k) $3.7 + 9 + 14.3 + \cdots + 30.2$

l) $2.84 + 5.3 + 7.76 + \cdots + 79.1$

2. Find the indicated value using the information given.

a) S_{20}, if $a_1 = 8$, $a_{20} = 65$

b) S_{21}, if $a_1 = 8$, $a_{20} = 65$

c) S_{56}, if $a_{56} = 13$, $d = -9$

d) n, if $S_n = 180$, $a_1 = 4$, $a_n = 16$

e) d, if $S_{40} = 680$, $a_1 = 11$

f) S_{62}, if $a_1 = 10$, $d = 3$

g) S_{19}, if $d = 4$, $a_{19} = 17$

h) S_{40}, if $d = -3$, $a_{40} = 65$

i) S_{40}, if $a_5 = 42$, $a_{15} = -18$

j) S_{20}, if $a_8 = 17$, $a_{15} = 38$

3. Find the indicated sum.

a) $\sum_{n=1}^{100} n$

b) $\sum_{k=100}^{200} k$

c) $\sum_{j=0}^{72} (3j-4)$

d) $\sum_{x=7}^{24} (2x+5)$

e) $\sum_{y=11}^{48} \left(\frac{y+4}{2}\right)$

f) $\sum_{z=51}^{100} (200-z) - \sum_{z=1}^{50} (200-z)$

4. Insert k arithmetic means between the given pair of numbers.

a) $5,\ 10,\ k=2$

b) $3,\ 6,\ k=3$

c) $a,\ b,\ k=2$

d) $a,\ b,\ k=3$

5. Solve for b: $\sum_{x=2}^{b} (23-2x) = 91$

6. Find the sum: $\sum_{x=a}^{b} 5$

7. What is the last element in the 20th row?

			1			
		2		3		
	4		5		6	
7		8		9		10

8. How many terms of the arithmetic series $1491 + 1484 + 1477 + \cdots$ are needed to give a sum of zero?

9. An auditorium has eight seats in the first row. Each subsequent row has four more seats than the previous row. How many seats are there in the 50th row of the auditorium?

10. If $1000 is deposited into the bank the day a child is born, and $100 more than the previous deposit is made each year until the child's 18th birthday, how much will be in the account, excluding interest?

11. Find the sum of all multiples of 6 between 50 and 500.

12. The sum of three consecutive terms of an arithmetic sequence is 3. The sum of their squares is 75. Find the three numbers.

13. If 20 people in a class shake hands with each other exactly once, how many handshakes will take place?

14. If the sum of the terms of an arithmetic series is 234, and the middle term is 26, find the number of terms in the series.

SOLUTIONS

6.5 Arithmetic Sequence

1. a) natural b) finite c) infinite d) recursive e) $t_n = a + (n-1)d$
2. a) $-1, 2, 7, 14$ b) $\frac{3}{2}, \frac{4}{3}, \frac{5}{4}, \frac{6}{5}$ c) $1, -4, 9, -16$ d) $1, \frac{9}{5}, 3, \frac{81}{17}$ e) $2, 1, \frac{8}{9}, 1$ f) $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}$
3. a) $\frac{1}{n}$ b) $\frac{1}{2^{n-1}}$ c) $\left(\frac{2}{3}\right)^n$ d) $(-1)^{n+1} \cdot 2n$
4. a) $4, 6, 8, 10$ b) $3, -1, 4, 0$ c) $2, 3, 5, 8$ d) $-1, 1, 2, 9$
5. a) 20 b) 22 c) 50 d) 32 e) 15 f) 194
6. a) $\sum_{k=1}^{4}(2k-1)$ b) $\sum_{k=1}^{5} k^2$ c) $\sum_{k=1}^{n} \frac{k}{k+1}$ d) $\sum_{k=1}^{n} \frac{5^k}{k}$
7. a) $7, 11, 15, 19, 23$ b) $15, 12, 9, 6, 3$ c) $4, 6, 8, 10, 12$ d) $-1, -4, -7, -10, -13$
   e) $-5, -\frac{23}{4}, -\frac{13}{2}, -\frac{29}{4}, -8$ f) $-\frac{2}{3}, -\frac{7}{15}, -\frac{4}{15}, -\frac{1}{15}, \frac{2}{15}$
8. a) 38 b) $-\frac{4}{3}$ c) $\frac{15}{4}$ d) -21.25 e) 1.2 f) -25.75
9. a) 13 b) 18 c) 44 d) 18 e) 33 f) 42
10. a) -5 b) -4 c) 47 d) 15 e) 21 f) $40\frac{1}{3}$
11. a) $-\frac{3}{2}$ b) 1 c) -4 d) -2 e) $-3, 4$ f) $-3, \frac{1}{2}$
12. d = difference
13. 1, 1, 2, 3, 5, 8, 13
14. $t = a + (n-1)d \rightarrow 50\,000 = 23\,750 + (n-1)(1250) \rightarrow 21 = n - 1 \rightarrow n = 22$ years
15. $t = a + (n-1)d \rightarrow 140 = 8 + (n-1)(4) \rightarrow 33 = n - 1 \rightarrow n = 34$, row 34
16. $t = a + (n-1)d \rightarrow t = 8 + (120-1)(0.75) \rightarrow t = 97.25$; \$97.25
17. $t = a + (n-1)d \rightarrow 24 = 8 + (n-1)\left(\frac{1}{4}\right) \rightarrow 64 = n - 1 \rightarrow n = 65$ days
    September has 30 days, October has 31 days, together they have 61 days. Therefore he died November 4th.
18. $\frac{(x-3)+(3x-11)}{2} = \frac{x^2}{25} + 9 \rightarrow x^2 - 50x + 400 = 0 \rightarrow (x-40)(x-10) = 0 \rightarrow x = 10, 40$
    $x = 10$: $7, 13, 19, 25$; $x = 40$: $37, 73, 109, 145$; The fourth term is 25 or 145.
19. $\frac{(2x-1)+(11-x^2)}{2} = x^2 - 3 \rightarrow 3x^2 - 2x - 16 = 0 \rightarrow (3x-8)(x+2) = 0 \rightarrow x = -2, \frac{8}{3}$
    $x = -2$: -5, ___, 1, ___, 7; $x = \frac{8}{3}$: $\frac{39}{9}$, ___, $\frac{37}{9}$, ___, $\frac{35}{9}$; The second term is -2 or $\frac{38}{9}$.

6.6 Arithmetic Series

1. a) $n^2 + 2n$ b) $\frac{3n^2 - 5n}{2}$ c) 1027 d) 1224 e) 24 500 f) $338\sqrt{5}$ g) 120 h) $-11\,703$ i) $\frac{531}{8}$ j) $-\frac{686}{3}$
   k) 101.7 l) 1311.04
2. a) 730 b) 798 c) 14 588 d) 18 e) $\frac{4}{13}$ f) 6293 g) -361 h) 4940 i) -2040 j) 490
3. a) 5050 b) 15 150 c) 7592 d) 648 e) 636.5 f) -2500
4. a) $\frac{20}{3}, \frac{25}{3}$ b) $\frac{15}{4}, \frac{9}{2}, \frac{21}{4}$ c) $\frac{2a+b}{3}, \frac{a+2b}{3}$ d) $\frac{3a+b}{4}, \frac{a+b}{2}, \frac{a+3b}{4}$
5. 8, 14
6. $-5a + 5b + 5$
7. Consider the last term in each row: $1, 3, 6, 10 \rightarrow 1, 1+2, 1+2+3, 1+2+3+4, \ldots, 1+2+\cdots+20$
   $S = \frac{n}{2}(2a + (n-1)d) \rightarrow \frac{20}{2}((2)(1) + (20-1)(1)) = 210$
8. $S = \frac{n}{2}(2a + (n-1)d) \rightarrow 0 = \frac{n}{2}((2)(1491) + (n-1)(-7)) \rightarrow 0 = 2982 + (n-1)(-7) \rightarrow 426 = n - 1 \rightarrow n = 427$
9. $t = a + (n-1)d \rightarrow t = 8 + (50-1)(4) \rightarrow t = 204$ seats
10. On the 18th birthday, the child has had 19 birthdays. $S = \frac{n}{2}(2a + (n-1)d) \rightarrow \frac{19}{2}((2)(1000) + (19-1)(100)) = \$36\,100$
11. The first multiple of 6 is 54. The last multiple of 6 is 498.
    $t = a + (n-1)d \rightarrow 498 = 54 + (n-1)(6) \rightarrow n = 75$; $S = \frac{n}{2}(a + l) \rightarrow S = \frac{75}{2}(54 + 498) = 20\,700$
12. $a + (a+d) + (a+2d) = 3 \rightarrow a + d = 1$
    $a^2 + (a+d)^2 + (a+2d)^2 = 75 \rightarrow (1-d)^2 + 1 + (1+d)^2 = 75 \rightarrow d^2 = 36 \rightarrow d = \pm 6 \rightarrow a = -5, 7$
    The first three numbers are $-5, 1, 7$ or $7, 1, -5$.
13. The first person shakes 19 hands, the second person shakes 18 hands, and so on.
    $19 + 18 + 17 + \cdots + 1 \rightarrow S = \frac{n}{2}(a + l) \rightarrow S = \frac{19}{2}(19 + 1) = 190$ handshakes
14. $S = \frac{n}{2}(a + l) = 234 \rightarrow n\left(\frac{a+l}{2}\right) = 234 \rightarrow 26n = 234 \rightarrow n = 9$ terms