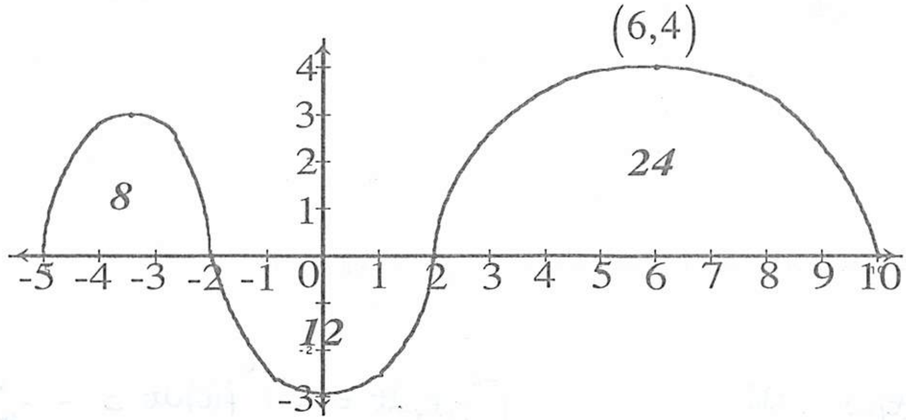
WS AP TOPICS REVIEW #1

NAME: Solutions



1) Let f be a function defined on the closed interval $-5 \le x \le 10$ with $f(2)=15$. The graph of f', the derivative of f, is shown above.

a) On what intervals, if any, is f increasing? Justify your answer.

f is increasing when f' is positive (or 0 a finite number of times)

$\Rightarrow$ on $[-5,-2]$ and $[2,10]$

b) On what intervals, if any, is f concave up? Justify your answer.

f is concave up when f' is increasing.

$\Rightarrow$ on $[-5,-3.5)$ and on $(0,6)$

c) Find the x-coordinate of each point of rel. max. of the graph of f on the open interval $-5 < x < 10$. Justify your answer.

f has a relative max when f' changes from + to −

$\Rightarrow$ when $x=-2$

d) Find an equation for the line tangent to the graph of f at the point (6,27).

slope: $f'(6)=4$
point $(6,27)$

$y-27=4(x-6)$ or $\boxed{y=4x+3}$

e) Find $f(-5)$ and $f(10)$. Show the work that leads to your answers.

$f(-5) = f(2) + \int_2^{-5} f'(t)\,dt = 15 - \int_{-5}^{2} f'(t)\,dt = 15-(8-12) = \boxed{19}$

$f(10) = f(2) + \int_2^{10} f'(t)\,dt = 15 + 24 = \boxed{39}$

f) Find the absolute maximum and minimum value from $[-5,10]$. Justify your answer.

x	-5		-2		2		10
$f'(x)$	0	+	0	−	0	+	0
$f(x)$	19	↗	27	↘	15	↗	39

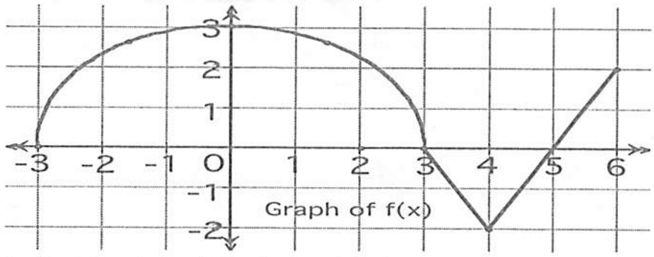
critical points: $-5, -2, 2, 10$

$f(-5) = 19$

$f(-2) = f(2) + \int_2^{-2} f'(t)\,dt = 27$

$f(2) = \boxed{15}$ ← abs min

$f(10) = \boxed{39}$ ← abs max



2) The graph of a function f consists of a semicircle and two line segments as shown above.

Let g be the function given by $g(x)=\int_0^x f(t)\,dt$.

a) Find each:

$g(3)= \int_0^3 f(t)\,dt = \frac{9}{4}\pi$

$g(-3)= \int_0^{-3} f(t)\,dt = -\int_{-3}^{0} f(t)\,dt = -\frac{9\pi}{4}$

$g(4)= \int_0^4 f(t)\,dt = \frac{9\pi}{4}-1$
$= \int_0^3 f(t)\,dt + \int_3^4 f(t)\,dt$

$g(6)= \int_0^6 f(t)\,dt = \int_0^3 f(t)\,dt + \int_3^4 f(t)\,dt + \int_4^5 f(t)\,dt + \int_5^6 f(t)\,dt$
$= \frac{9\pi}{4}-1$

b) Find each: $g'(4)= f(4) = -2$

$g'(x)=f(x)$ (FTC)

$g'(5)= f(5) = 0$

$g''(5)= f'(5) = 2$

c) Write an equation for the line tangent to the graph of g at $x=4$.

slope : $g'(4)=f(4)=-2$
point $(4, g(4))$ i.e $(4, \frac{9\pi}{4}-1)$

$\boxed{y-\frac{9\pi}{4}+1=-2(x-4)}$

d) For what values of x in the open interval $(-3,6)$ is the graph of g concave down? Explain your reasoning.

g is concave down when g' (i.e f) is decreasing.

$\Rightarrow$ on $(0,4)$

e) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-3,6)$. Justify your answer.

point of inflection when g' (i.e f) changes direction

$\Rightarrow$ when $x=0$ and $x=4$

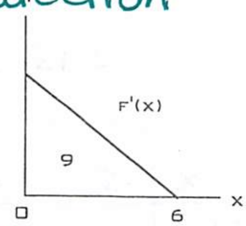
3) If the velocity of the particle is 100 at $t=3$ and the acceleration of the particle is $a(t)=11\sin t$, find the velocity of the particle at $t=7$.

$v(7) = v(3) + \int_3^7 a(t)\,dt$

$= 100 + 11\,(-\cos t)\Big]_3^7$

$= 100 - 11\cos 7 + 11\cos 3$

4) If $f(6)=50$
Find $f(0)=$



$f(6)-f(0) = \int_0^6 f'(t)\,dt$

$f(0) = 50 - 9$

$= 41$